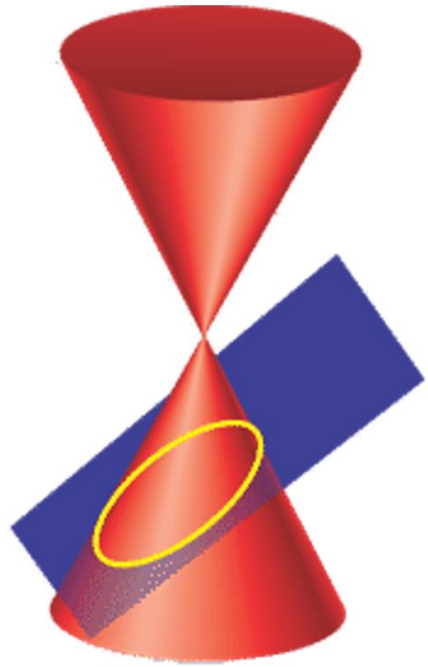
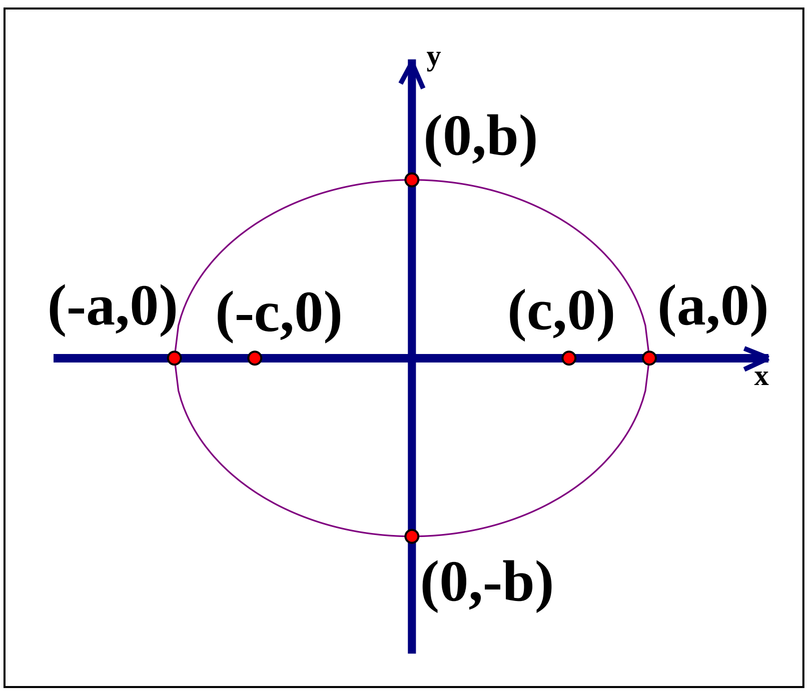
Ellipses

Objective 1: Equations and Graphs of Ellipses

An ellipse is one of the conic sections. A conic section is the intersection of a plane with a right circular cone.

Definition: An **ellipse** is a set of points in a plane, the sum of whose distances from two fixed points is a constant. The fixed points are called **foci**.





$$\sqrt{(x-(-c))^2+(y-0)^2}+\sqrt{(x-c)^2+(y-0)^2}=2a$$

$$\left(\sqrt{(x+c)^2+y^2}\right)^2=\left[2a-\sqrt{(x-c)^2+y^2}\right]^2$$

$$x^2+2cx+c^2+y^2=4a^2-4a\sqrt{(x-c)^2+y^2}+x^2-2cx+c^2+y^2$$

$$4a\sqrt{(x-c)^2+y^2}=4a^2-4cx$$

$$\left(a\sqrt{x^2-2cx+c^2+y^2}\right)^2=\left(a^2-cx\right)^2$$

$$a^2\left(x^2-2cx+c^2+y^2\right)=a^4-2a^2cx+c^2x^2$$

$$a^2x^2-2a^2cx+a^2c^2+a^2y^2=a^4-2a^2cx+c^2x^2$$

$$\left(a^2-c^2\right)x^2+a^2y^2=a^4-a^2c^2$$

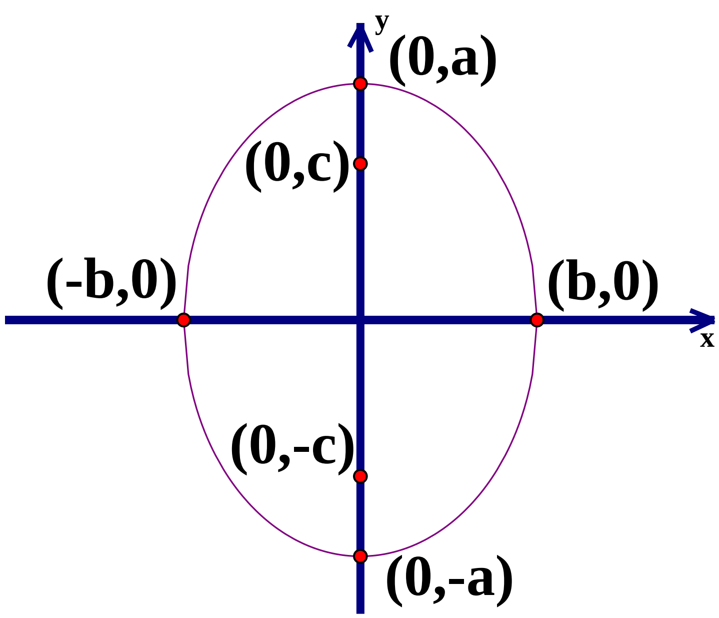
$$\frac{\left(a^2-c^2\right)x^2}{a^2(a^2-c^2)}+\frac{a^2y^2}{a^2(a^2-c^2)}=\frac{a^4-a^2c^2}{a^2(a^2-c^2)}$$

$$\frac{x^2}{a^2}+\frac{y^2}{(a^2-c^2)}=1$$

$Let\ b^2=a^2-c^2$

$\left(so\ \ a^2=b^2+c^2\right)$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$



$$\frac{y^2}{a^2}+\frac{x^2}{b^2}=1$$

For ellipses:

$a > b$ and $a^2 = b^2 + c^2$

To translate to the center (h,k) the equations become:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \quad \text{and}$$

$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$

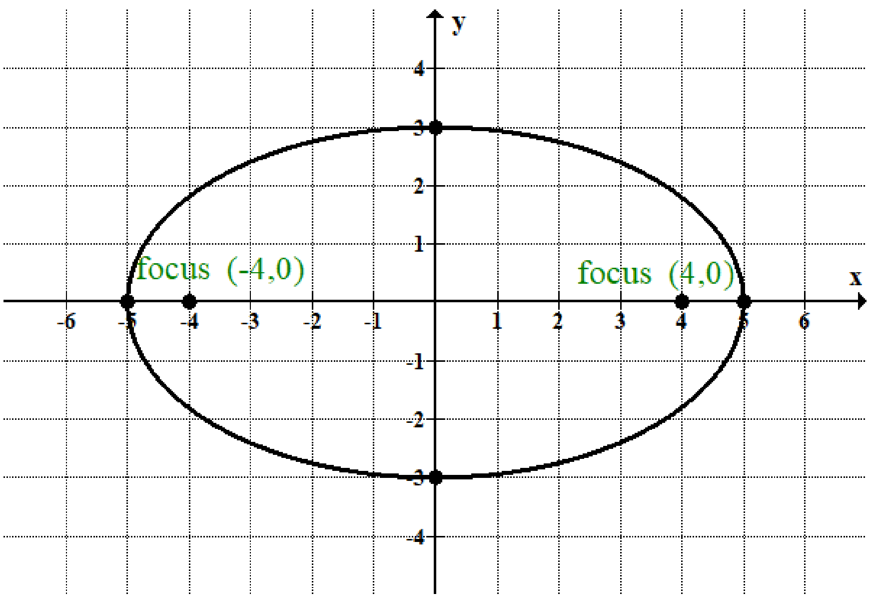
a is the distance from the center to the vertex.

c is the distance from the center to the focus.

b is the distance from the center to the endpoints of the minor axis (co-vertices).

a, b and c are all positive quantities.

Example: Match the graph to its equation.



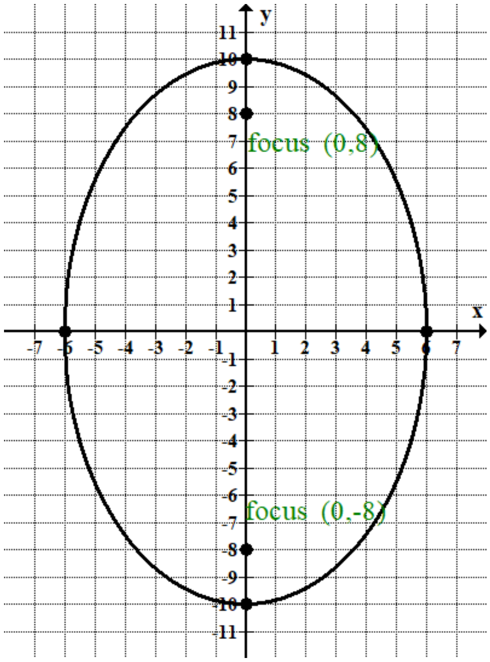
A. $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$

B. $\dfrac{x^2}{9} + \dfrac{y^2}{25} = 1$

C. $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$

D. $\dfrac{y^2}{25} + \dfrac{x^2}{9} = 1$

Example: Match the graph to its equation.



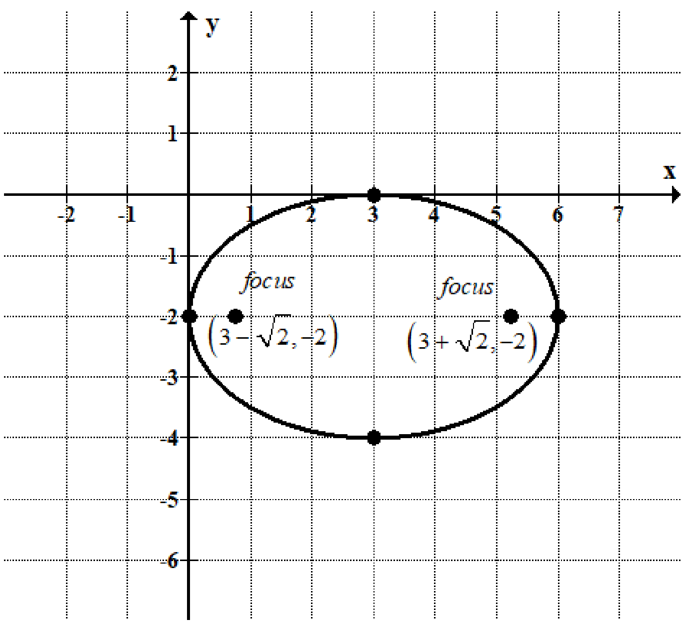
A. $\dfrac{x^2}{6}+\dfrac{y^2}{10}=1$

B. $\dfrac{x^2}{36}+\dfrac{y^2}{100}=1$

C. $\dfrac{y^2}{36}+\dfrac{x^2}{100}=1$

D. $\dfrac{x^2}{10}+\dfrac{y^2}{6}=1$

Example: Match the graph to its equation.



A. $\dfrac{(x-3)^2}{9}+\dfrac{(y+2)^2}{4}=1$

B. $\dfrac{(x-3)^2}{3}+\dfrac{(y+2)^2}{2}=1$

C. $\dfrac{(y-3)^2}{9}+\dfrac{(x+2)^2}{4}=1$

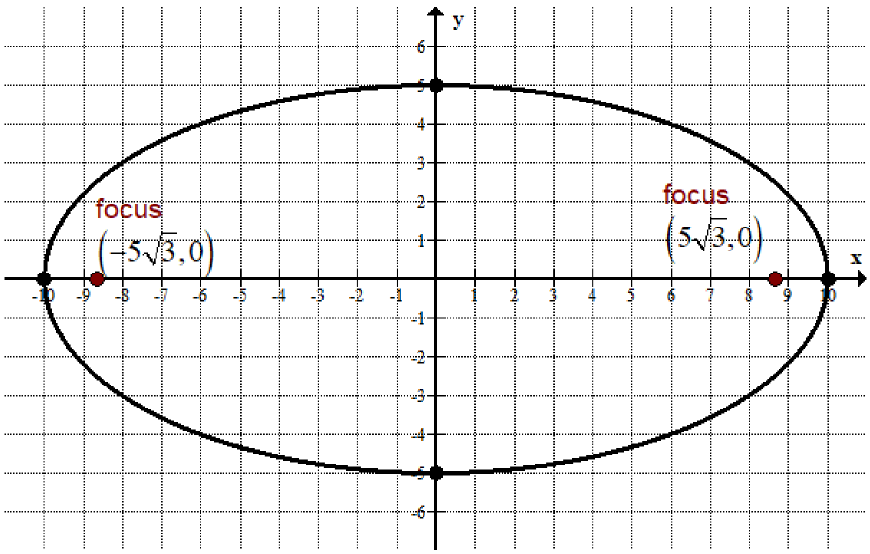
D. $\dfrac{(x-3)^2}{4}+\dfrac{(y+2)^2}{9}=1$

Pause the video to try this one on your own, then restart when you are ready to check your answer.

Extra Practice

Match the graph to its equation.

1. ______



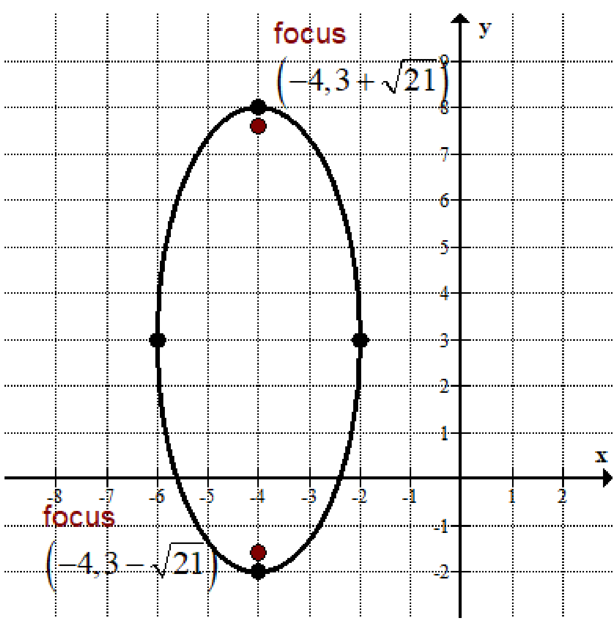
A. $\dfrac{x^2}{25}+\dfrac{y^2}{100}=1$

B. $\dfrac{x^2}{10}+\dfrac{y^2}{5}=1$

C. $\dfrac{x^2}{100}+\dfrac{y^2}{25}=1$

D. $\dfrac{x^2}{5}+\dfrac{y^2}{10}=1$

2. ______



A. $\dfrac{(x+4)^2}{4}+\dfrac{(y-3)^2}{25}=1$

B. $\dfrac{(x+4)^2}{25}+\dfrac{(y-3)^2}{4}=1$

C. $\dfrac{(x-3)^2}{4}+\dfrac{(y+4)^2}{25}=1$

D. $\dfrac{(x-3)^2}{25}+\dfrac{(y+4)^2}{4}=1$

Restart when you are ready to check your answers.

Objective 2: Convert from General Form to Standard Form

General form for the equation of a conic section:

$$Ax^2 + By^2 + Cx + Dy + F = 0$$

Standard form for the equation of an ellipse:

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \quad or$$

$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$

Example: Convert the following equation to standard form. Identify the center, vertices and foci.

$$25x^2 + 4y^2 + 100x - 24y + 36 = 0$$

Example: Convert the following equation to standard form. Identify the center, vertices and foci.

$$y^2 + 25x^2 - 6y - 250x + 633 = 0$$

Pause the video to try this one on your own, then restart when you are ready to check your answer.

Extra Practice

Questions:

1. Convert the following equation to standard form. Identify the center, vertices and foci.

   $$9x^2 + 4y^2 - 18x + 16y - 11 = 0$$

2. Convert the following equation to standard form. Identify the center, vertices and foci.

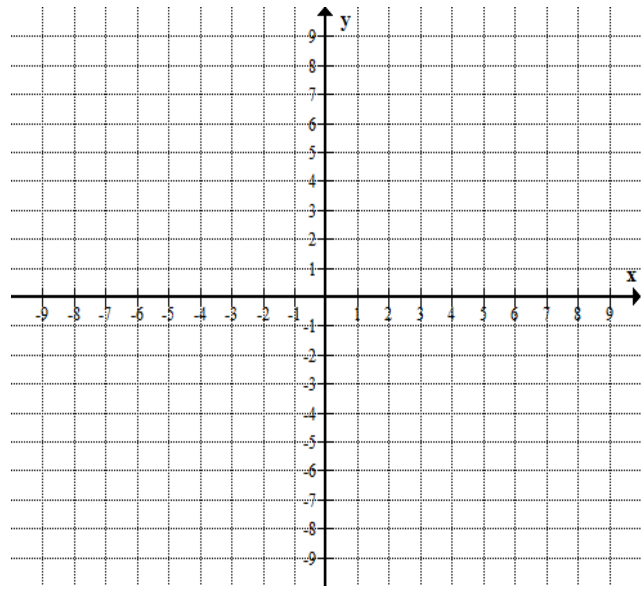
   $$x^2 + 9y^2 + 6x - 18y + 9 = 0$$

Restart when you are ready to check your answers.

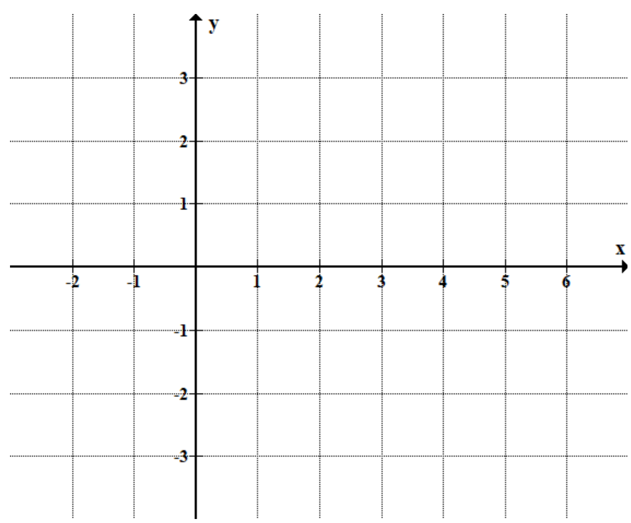
Objective 3: Graph Ellipses

When you graph an ellipse, include the vertices, endpoints of the minor axis, the foci and their exact coordinates.

Example: Graph $\frac{(x+2)^2}{16}+\frac{(y-3)^2}{25}=1$



Example: Graph $\frac{(x-2)^2}{9}+\frac{y^2}{7}=1$

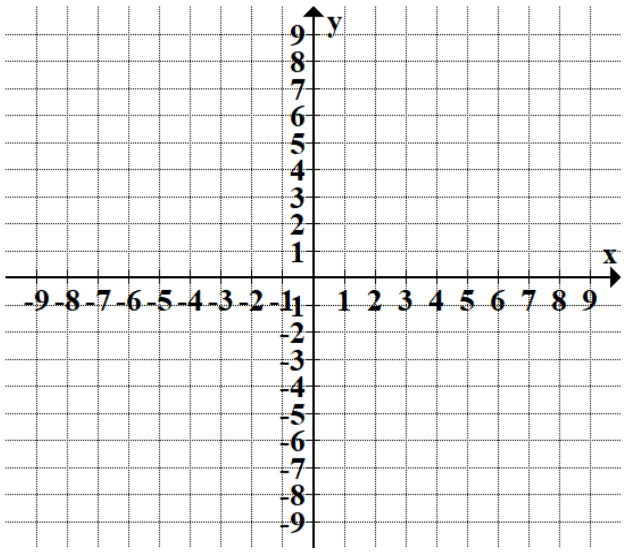


Pause the video to try this one on your own, then restart when you are ready to check your answer.

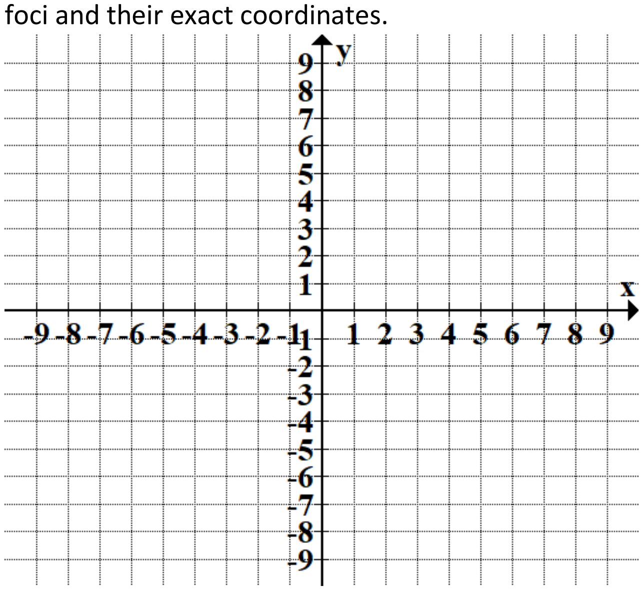
Extra Practice

Questions:

1. Graph $\frac{(y+3)^2}{36}+\frac{(x-1)^2}{25}=1$. Make sure include the vertices, endpoints of the minor axis, foci and their exact coordinates.



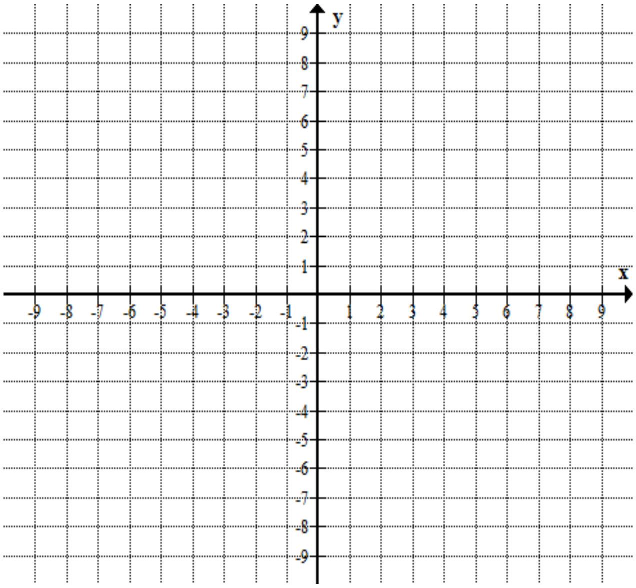
2. Graph $\frac{(y-2)^2}{4}+\frac{(x+5)^2}{9}=1$. Make sure include the vertices, endpoints of the minor axis, foci and their exact coordinates.



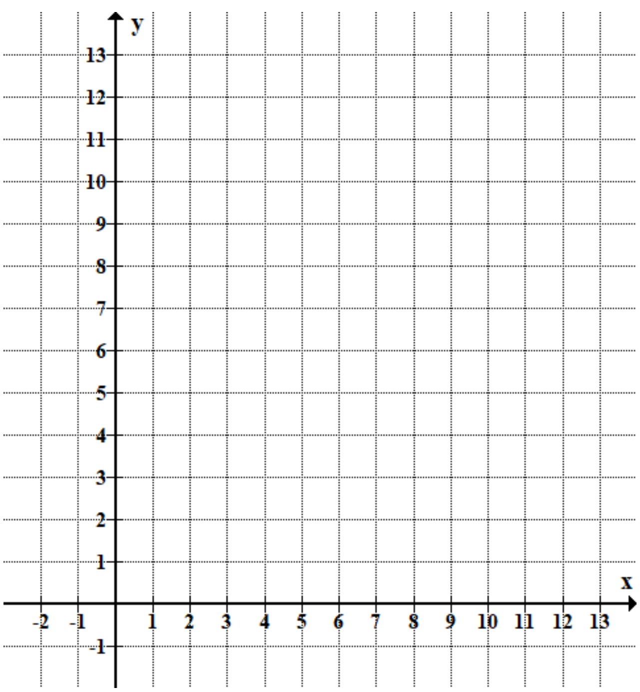
Restart when you are ready to check your answers.

Objective 4: Determine the Equation of an Ellipse

Example: Determine the equation of an ellipse with foci at (-3,2) and (-3,-6) and a vertex at (-3,-8). Write your answer in standard form.



Example: Find an equation in standard form for the ellipse with center at (8, 9), horizontal minor axis of length 8 and passes through the point (11, 6).

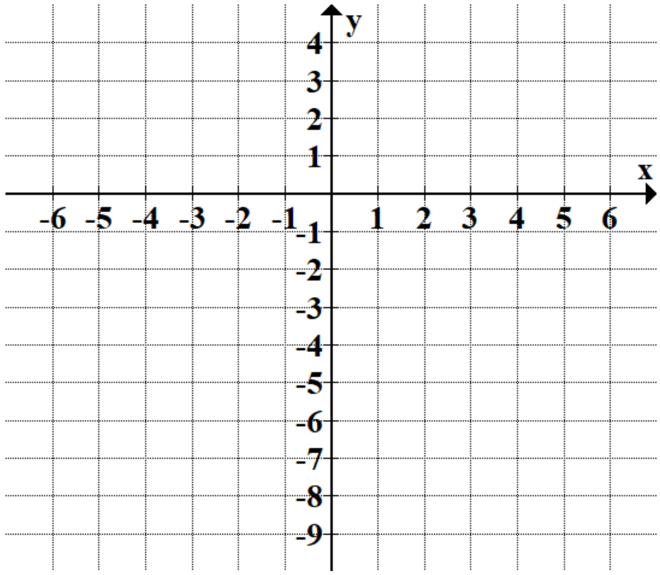


Pause the video to try this one on your own, then restart when you are ready to check your answer.

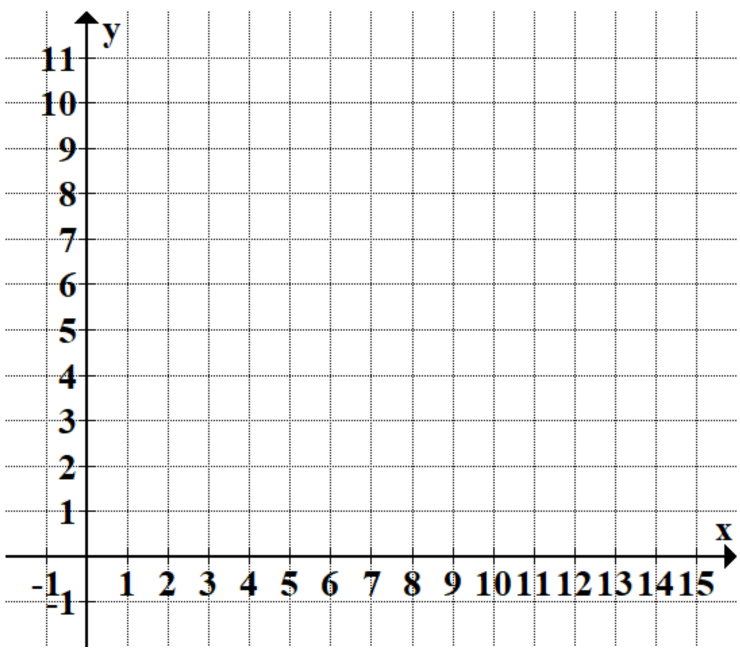
Extra Practice

Questions:

1. Determine the equation of an ellipse with foci at (0,-1) and (0,-7) and a vertex at (0,-9). Write your answer in standard form.



2. Find an equation in standard form for the ellipse with center at (9, 6), horizontal major axis of length 10 and passes through the point (13, 5).



Restart when you are ready to check your answers.

Objective 5: Applications of Ellipses

Example: A certain bridge arch is in the shape of half an ellipse 112 feet wide and 29.5 feet high. At what horizontal distance from the center of the arch is the height equal to 13.7 feet? Round your answer to one decimal place.

Example: A person in a whispering gallery standing at one focus of the ellipse can whisper and be heard by a person standing at the other focus because all the sound waves that reach the ceiling are reflected to the other person. If a whispering gallery has a length of 150 feet, and the foci are located 20 feet from the center, find the height of the ceiling at the center. Round your answer to 4 decimal places.

Pause the video to try this one on your own, then restart when you are ready to check your answer.

Extra Practice

Questions:

1. A certain bridge arch is in the shape of half an ellipse 128 feet wide and 26.8 feet high. At what horizontal distance from the center of the arch is the height equal to 16.4 feet? Round your answer to one decimal place.

2. A person in a whispering gallery standing at one focus of the ellipse can whisper and be heard by a person standing at the other focus because all the sound waves that reach the ceiling are reflected to the other person. If a whispering gallery has a length of 150 feet, and the foci are located 20 feet from the center, find the height of the ceiling at the center. Round your answer to 4 decimal places.

Restart when you are ready to check your answers.