Factoring Trinomials – Basics (with $a = 1$)

Objective 1: Identify polynomial, monomial, binomial, trinomial, and the degree of the polynomial

Let us classify different types of the polynomials:

Monomial: a polynomial with exactly one term

Binomial: a polynomial with exactly two terms

Trinomial: a polynomial with exactly three terms

Degree of a polynomial in one variable is the **largest exponent** of the variable.

For example: $5x^7$: ______________________________

$3x + 15$: ______________________________

$7x^2 + 5x - 3$: ______________________________

$4x^3 - 6x^2 + 2x + 9$: ______________________________

$2x^5 + 5x^3 - 6x$: ______________________________

EXERCISE: Pause the video and try these problems.

Ex) Classify the type of each polynomial and give the degree of the polynomial.

1. $5x^3$ ________________________________
2. $2x^6 + 5x^3 - 7$ ________________________________
3. $4x^7 - 9x^4$ ________________________________
4. $x^5 + 2x^4 - 7x^3 - 5x$ ________________________________

Objective 2: Find and factor out the GCF from the terms of a polynomial

Consider the following examples:

Multiplying: $5x^3(x^2+3) = 5x^5 + 15x^3$

Factoring: $5x^5 + 15x^3 = 5x^3(x^2+3)$

Do you see that factoring is the reverse process of multiplying?

Factoring

$$5x^5 + 15x^3 = \mathbf{5x^3}(x^2+3)$$

Multiplying

Note: $\mathbf{5x^3}$ is called the GCF.

The first step in factoring a polynomial is to find and factor out the GCF (Greatest Common Factor).

Finding the Greatest Common Factor (GCF)

Step 1: ***Factor.*** Write each term in prime factored form.
Step 2: ***List common factors.*** List each prime factor that is common in every term in the list.
Step 3: ***Choose least exponent.*** Use the smallest exponent of each common prime factor.
Step 4: ***Multiply.*** Multiply the primes from Step 3. If there are no primes left at Step 3, the GCF is 1.

Example: Factor out the GCF: $8x^5y^3 + 24x^4 - 20x^3y^4$

$8x^5y^3 =$

$24x^4 =$

$20x^3y^4 =$

GCF=

Factor out the GCF of the polynomial:

$$8x^5y^3 + 24x^4 - 20x^3y^4 =$$

EXERCISE: Pause the video and try these problems.

Ex) Factor out the Greatest Common Factor (GCF).

1. $6x^3y^2 + 10xy^2$

2. $20x^5y^3 - 15x^4y + 10x^3y^2$

3. $12x^7y^3 + 9x^4y^4 + 3x^3y^2$

4. $8x^3y^3z^2 + 12x^4y^3z - 20x^4y^2$

Objective 3: Factor trinomials of degree 2 of the form $\boldsymbol{x^2 + bx + c}$ $(\boldsymbol{a = 1})$

Consider the following examples:

Multiplying: $(x+2)(x+3) = x^2 + 5x + 6$

The product of these numbers is 6.

Factoring: $x^2 + 5x + 6 = (x+2)(x+3)$

The sum of these numbers is 5.

To factor $x^2 + 5x + 6$, we want to find two integers whose product is 6 and whose sum is 5.

To factor $x^2 + bx + c$, we want to find two integers whose product is c and whose sum is b.

EXERCISE: Pause the video and try these problems.

1. Find two integers whose product is 36 and whose sum is 13. ____________________
2. Find two integers whose product is -6 and whose sum is -5. ____________________
3. Find two integers whose product is 40 and whose sum is -13. ____________________
4. Find two integers whose product is -24 and whose sum is 5. ____________________
5. Find two integers whose product is 10 and whose sum is -3. ____________________

To Factor a Trinomial of the Form $\boldsymbol{x^2 + bx + c}$:

Find two integers whose product is $\boldsymbol{c}$ and whose sum is $\boldsymbol{b}$ and then write in binomial factors:

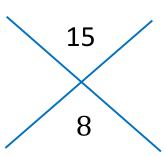
The product of these numbers is c.

$$x^2 + bx + c = (x + \square)(x + \square)$$

The sum of these numbers is b.

<u>Example:</u> Factor each trinomial. ("X-Box" method)

Ex) $x^2 + 8x + 15$



Ex) $x^2 - 9x + 14$

Ex) $x^2 + 2x - 24$

Ex) $x^2 - 4xy - 45y^2$

EXERCISE: Pause the video and try these problems.

Ex) Factor each trinomial.

1. $x^2 + 10x + 21$
2. $x^2 - x - 30$
3. $x^2 - 12x + 32$
4. $x^2 - 2x + 15$
5. $x^2 + 10xy - 24y^2$

Objective 4: Factor trinomials completely

Factoring trinomials completely

Step 1: If the leading coefficient is negative, factor out the negative sign.
Step 2: If there are any common factors, factor out the GCF.
Step 3: Factor the resulting trinomial if possible.

Example: Factor completely: $-4x^3y^2 - 8x^2y^2 + 60xy^2$

EXERCISE: Pause the video and try these problems.

Ex) Factor each polynomial completely.

1. $3x^4 - 3x^3 - 60x^2$
2. $2x^4y^2 + 14x^3y^2 + 20x^2y^2$
3. $-5x^3y^2 + 5x^2y^2 + 60xy^2$
4. $-4x^5 + 20x^4y - 24x^3y^2$