

# Functions - Intermediate

## Objective 1: Definition of a Function

Let's say the cost of hamburger is \$3 per pound.  
If we buy 1 pound it will cost us  $\$3(1) = \$3$ .  
If we buy 2 pounds it will cost us  $\$3(2) = \$6$ .  
If we buy 3 pounds it will cost us  $\$3(3) = \$9$ .

Now, let  $x$  represent the number of pounds of hamburger purchased.  
Then the cost will be  $\$3(x) = \$3x$ .  
Let  $y$  represent the cost.  
Then the equation  $y = 3x$  relates the cost in dollars,  $y$ , to the number of pounds,  $x$ .

$x$	$y$
1	3
2	6
3	9

$(x,y)$  is an ordered pair.  
 $x$  and  $y$  are the components.

A **relation** is any set of ordered pairs. It is a correspondence or mapping between two sets.  
The set of all first components is the domain.  
The set of all second components is the range.

The domain is the weight of the hamburger purchased.  
The range is the price paid.

**Example:** In the following relation, identify the domain and the range.

$\{(3,5),(7,11),(10,2),(12,15)\}$

A **function** is a set of ordered pairs where each first component is paired with exactly one second component.

**Example:** Consider the following relation:  $\{(0,2),(-1,3),(2,3),(4,5)\}$

- Is the relation a function?
- What is the domain?
- What is the range?

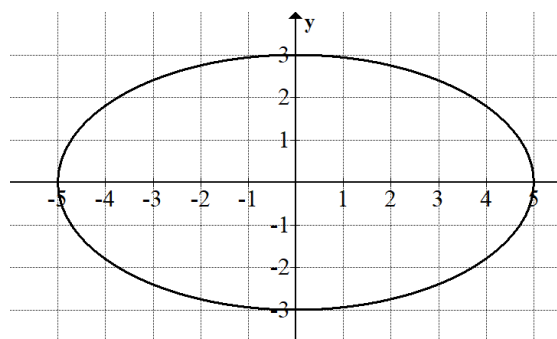
**Example:** Consider the following relation:  $\{(5,7),(9,-2),(9,4),(10,6)\}$

- Is the relation a function?
- What is the domain?
- What is the range?

If a vertical line can pass through more than one point on the graph of a relation, then the relation is not a function. This is called the **vertical line test**.

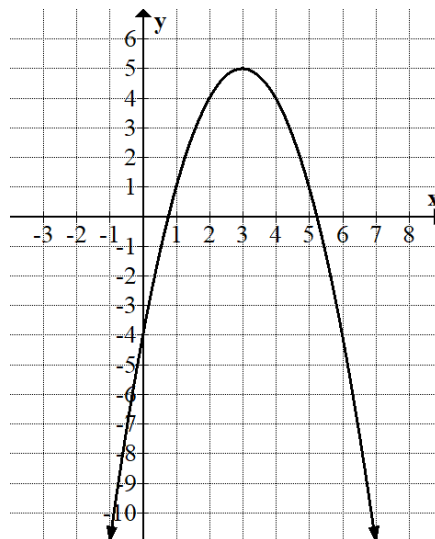
**Example:** Consider the following graph of a relation:

- Is the relation a function?
- What is the domain?
- What is the range?



**Example:** Consider the following graph of a relation:

- a) Is the relation a function?
- b) What is the domain?



- c) What is the range?

Pause the video to try this one on your own, then restart when you are ready to check your answer.

### Extra Practice

1. Consider the following relation:  $\{(5,8), (7,5), (9,3), (11,5)\}$ 
  - a) Is the relation a function?
  - b) What is the domain?
  - c) What is the range?
  
2. Consider the following relation:  $\{(-6,4), (3,-7), (-5,1), (-5,9)\}$ 
  - a) Is the relation a function?
  - b) What is the domain?
  - c) What is the range?

Restart when you are ready to check your answers.

## Objective 2: Find Values of a Function

**Example:** Let  $f(x) = 3x - 5$ , find

a)  $f(4)$

b)  $f(-2)$

c)  $f(0)$

**Example:** Let  $f(x) = -x^2 - 4x + 2$ , find

a)  $f(0)$

b)  $f(-3)$

c)  $f(5)$

**Example:** Let  $f(x) = \frac{4-x}{x^2+5}$ , find

a)  $f(0)$

b)  $f(2)$

c)  $f(-6)$

**Pause the video to try this one on your own, then restart when you are ready to check your answer.**

### Extra Practice

1) Let  $f(x) = 7 - 3x$

a)  $f(5)$

b)  $f(-2)$

2) Let  $f(x) = -x^2 + 4x - 3$

a)  $f(3)$

b)  $f(-4)$

3) Let  $f(x) = \frac{x-4}{x^2-5}$

a)  $f(4)$

b)  $f(-3)$

Restart when you are ready to check your answers.

### Objective 3: Functions Evaluated at a Constant

**Example:** Let  $f(x) = 7 - 2x$ , find

a)  $f(a)$

b)  $f(3) + f(2)$

c)  $f(5) - f(a)$

d)  $f(x+3)$

**Example:** Let  $f(x) = -3x^2 - 4x + 7$ , find

a)  $f(a)$

b)  $f(5) + f(-2)$

c)  $f(-7) - f(a)$

d)  $f(x-4)$

**Example:** Let  $f(x) = \frac{9-x}{x+3}$ , find

a)  $f(a)$

b)  $f(x-5)$

Pause the video to try this one on your own, then restart when you are ready to check your answer.

### Extra Practice

1. Let  $f(x) = 7 - 2x$ , find

a)  $f(a)$



b)  $f(8) + f(-6)$

c)  $f(x-5)$

2. Let  $f(x) = 5x - 7 - 2x^2$ , find

a)  $f(4) + f(-3)$

b)  $f(5) - f(a)$

c)  $f(x-7)$

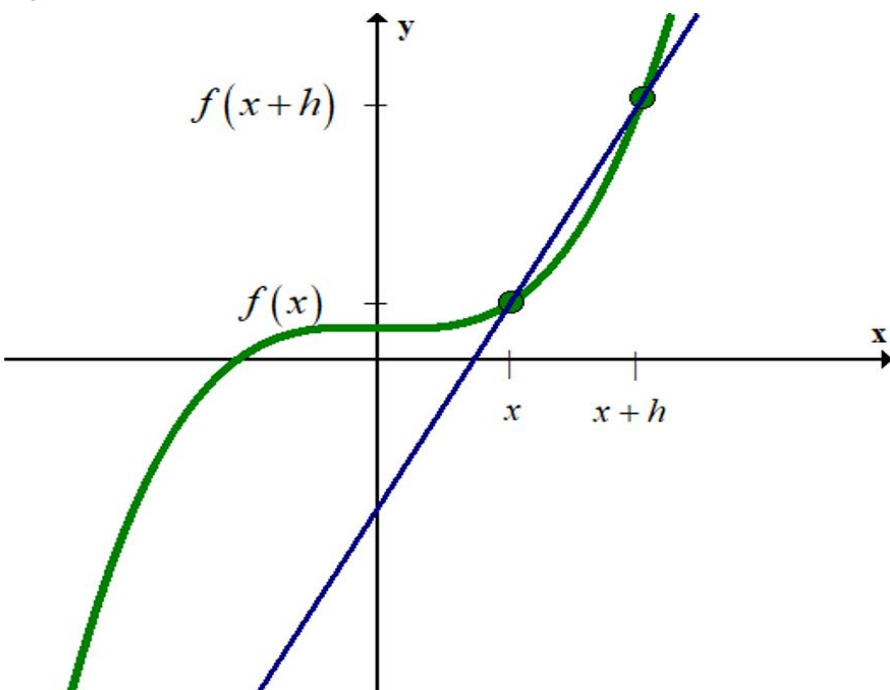
3. Let  $f(x) = \frac{3+x}{7-x}$ , find

a)  $f(a)$

b)  $f(x-4)$

Restart when you are ready to check your answers.

Objective 4: Find Difference Quotients



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\begin{aligned} & \frac{f(x+h) - f(x)}{(x+h) - x} \\ &= \frac{f(x+h) - f(x)}{h} \end{aligned}$$

**Example:** Let  $f(x) = -5x + 7$ , find

a)  $f(x+h)$

b)  $f(x+h) - f(x)$

c)  $\frac{f(x+h) - f(x)}{h}$

**Example:** Let  $f(x) = x^2 + 3$ , find

a)  $f(x+h)$

b)  $f(x+h) - f(x)$

c)  $\frac{f(x+h) - f(x)}{h}$

**Example:** Let  $f(x) = -3x^2 - x - 8$ , find

a)  $f(x+h)$

b)  $f(x+h) - f(x)$

c)  $\frac{f(x+h) - f(x)}{h}$

Pause the video to try this one on your own, then restart when you are ready to check your answer.

### Extra Practice

1. Let  $f(x) = 8 - 3x$ , find

a)  $f(x + h)$

b)  $f(x + h) - f(x)$

2. Let  $f(x) = 9x - 4$ , find  $\frac{f(x + h) - f(x)}{h}$

3. Let  $f(x) = 3x^2 - 4x - 7$ , find

c)  $f(x + h)$

d)  $f(x + h) - f(x)$

4. Let  $f(x) = -x^2 + 5x - 9$ , find  $\frac{f(x+h) - f(x)}{h}$

Restart when you are ready to check your answers.

### Objective 5: Difference Quotients for Rational and Radical Functions

**Example:** Let  $f(x) = \frac{5}{x}$ , find

a)  $f(x+h)$

b)  $\frac{f(x+h) - f(x)}{h}$

**Example:** Let  $f(x) = \frac{-7}{x^2}$ , find

a)  $f(x+h)$

b)  $\frac{f(x+h) - f(x)}{h}$

**Example:** Let  $f(x) = \sqrt{x}$ , find

a)  $f(x+h)$

b)  $\frac{f(x+h) - f(x)}{h}$

**Example:** Let  $f(x) = \sqrt{5x}$ , find

a)  $f(x+h)$

b)  $\frac{f(x+h) - f(x)}{h}$



Pause the video to try this one on your own, then restart when you are ready to check your answer.

### Extra Practice

1. Let  $f(x) = \frac{-9}{x}$ , find  $\frac{f(x+h) - f(x)}{h}$ .

2. Let  $f(x) = \frac{5}{x^2}$ , find  $\frac{f(x+h) - f(x)}{h}$ .

3. Let  $f(x) = \sqrt{3x}$ , find  $\frac{f(x+h) - f(x)}{h}$ .

Restart when you are ready to check your answers.