Linear Inequalities

Objective 1: Writing Interval and Set-builder Notation

There are different ways to writing a solution to an inequality.

Given the inequality: $x > 2$

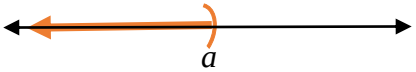
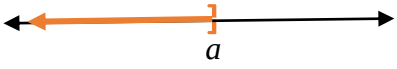
We can **graph** the solutions on a real number line:

We can use **interval notation**: $(2, \infty)$

We can also use **set-builder notation**: $\{x \mid x > 2\}$

This is read "The set of all x such that x is greater than two."

Here is a quick chart to help you distinguish between the 3 types of writing the solutions to the inequality.

Inequality	**Graph**	**Interval Notation**	**Set-Builder Notation**
$x > a$		(a, ∞)	$\{x \mid x > a\}$
$x \geq a$		$[a, \infty)$	$\{x \mid x \geq a\}$
$x < a$		$(-\infty, a)$	$\{x \mid x < a\}$
$x \leq a$		$(-\infty, a]$	$\{x \mid x \leq a\}$

Note: Always use a parenthesis when you use ∞ or $-\infty$.

Example: Graph the inequality and then write the solutions in interval notation and in set-builder notation.

$x \leq -4$

Graph: 

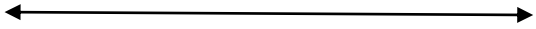
Interval notation:

- Notice when you *include* the endpoint in interval notation, you use a bracket.

Set-builder notation:

Example: Graph the inequality and then write the solutions in interval notation and in set-builder notation.

$x > 5$

Graph: 

Interval notation:

- When you write the interval notation that does NOT include the endpoint, use parentheses.

Set-builder notation:

Exercises

Given the inequality, write the correct set-builder and interval notation that represents the solution.

1. $12 < x$

2. $x \geq -5$

3. $x < 0$

4. $x \leq -11$

Objective 2: Solving Linear Inequalities with Variables on One Side

When finding the solutions to an inequality, solve for the variable like you would as if it were an equation.

Just remember when you divide or multiply by a negative number, you need to FLIP the inequality.

Example: Solve, graph the solution set, and write the solution in interval and set-builder notation.

$$2x - 8 > -4$$

Graph: 

Interval notation:

Set-builder notation:

Example: Solve, graph the solution set, and write the solution in interval and set-builder notation.

$-3p - 4p \geq 28$

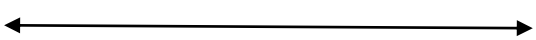
Graph: 

Interval notation:

Set-builder notation:

Example: Solve, graph the solution set, and write the solution in interval and set-builder notation.

$-18 < 5k + 4k$

Graph: 

Interval notation:

Set-builder notation:

Exercises

For each problem, graph the solution and write the solution using interval and set-builder notation.

1. $4x - 7 \geq 7$

2. $9x - 1 + 3x < 8 - 3$

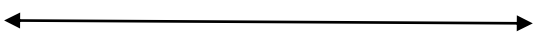
3. $x - 1 - 5x > -7$

4. $-7 - 3 \leq 2x - 14x + 2$

Objective 3: Solving Linear Inequalities with Variables on Both Sides

Example: Solve, graph the solution set, and write the solution in interval and set-builder notation.

$$-5x - 3 > 2x + 1$$

Graph: 

Interval notation:

Set-builder notation:

Example: Solve, graph the solution set, and write the solution in interval and set-builder notation.

$$y - 6 \leq 15 + 8y$$

Graph: 

Interval notation:

Set-builder notation:

Exercises

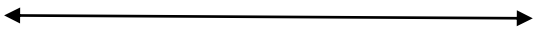
For each problem, graph the solution and write the solution using interval and set-builder notation.

1. $6x - 3 - 8 > 7x - 3x + 5$
2. $\frac{5x-1}{3} > 4x + 8$
3. $-x - 6 - 12 \leq 5x$
4. $3x - 9 \geq 14x - 13$

Objective 4: Solving Linear Inequalities by Using the Distributive Property

Example: Solve, graph the solution set, and write the solution in interval and set-builder notation.

$$5(7 + 2x) \geq 8 - x$$

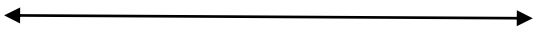
Graph: 

Interval notation:

Set-builder notation:

Example: Solve, graph the solution set, and write the solution in interval and set-builder notation.

$$8 - (4x - 2) < -5(x + 1) - 5$$

Graph: 

Interval notation:

Set-builder notation:

Example: Solve, graph the solution set, and write the solution in interval and set-builder notation.

$3(x-5) \leq 2(2x-1)$

Graph: 

Interval notation:

Set-builder notation:

Exercises

For each problem, graph the solution and write the solution using interval and set-builder notation.

1. $6(2x-3) \geq 4(5x-3)$
2. $\frac{1}{3}(4x-1) < \frac{1}{2}(3x+2)$
3. $8-3x-x < 2(x-6)$
4. $\frac{4}{5}(x-7) \leq -2(x-3)$