Performing Hypothesis Tests for One Population Mean and Standard Deviation

Introduction: The Idea of Hypothesis Testing

In a random sample of 500 Math professors, 276 of them believe (more like are telling themselves) that their students truly enjoy their Math classes. The idea of hypothesis testing is to answer the following questions: Does this sample result suggest that **more than 50%** of these professors' students truly enjoy their Math classes? Or is it possible that the percentage is only 50%, and we just happened to survey a group of students who enjoy their Math classes more than the population?

Hypothesis testing allows us to see whether it would it be ___________________to get the sample results of $\hat{p} = \frac{x}{n} = \frac{276}{500} = 0.552$ **or higher** from a population whose proportion is 0.5. From that point, we are able to evaluate whether the sample results are ***statistically significant***.

- ________________________________ – sample results that are ___________________________ given that the null hypothesis is true, keeping in mind that the null hypothesis is initially assumed to be true.
  - If sample results are statistically significant, we have _________________________ evidence to reject the null hypothesis.

Explanation of the *P*-value Approach:

- ________________________________ - the process of finding the _________________________ of obtaining found sample results ***under the assumption*** that the null hypothesis is true.
  - In other words, it is the probability that random chance could explain the sample results
  - If the probability of obtaining found sample results is ___________________ under the assumption that the null hypothesis is true, this is evidence that random chance cannot explain the sample results; and we use this as evidence to reject the null hypothesis.

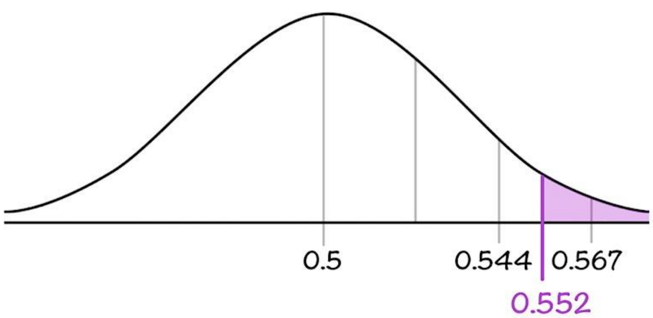
<u>Back to the example</u>: In a random sample of 500 Math professors, 276 of them believe (more like are telling themselves) that their students truly enjoy their Math classes. Test the claim that more than 50% of these professors' students truly enjoy their Math classes.

First, our sample results give us $\hat{p} = \frac{276}{500} =$ ______________

We need to find the __________________________of getting a sample proportion of 0.552 __________ __________________ ***under the assumption*** that the true population proportion is 0.5.

Since we (of course) remember that probability is simply the _______________________ under the curve, we can use the ***normalcdf*** function. Note: we will explain later how we know the curve is a normal curve.

Find the area under the curve as displayed by the figure.



The area under the curve is ____________________, which is the **probability** that we would get a $\hat{p} = 0.552$ ***under the assumption*** that the null hypothesis (p = 0.5) is true.

Well, we have a *P*-value = ______________. That is, the probability of getting sample result like we did by random chance is only _________, which is definitely ____________________.

Thus, since the probability of getting a sample result like this is so small, we use this as evidence to ___________________the null hypothesis.

Objective 1 – Testing Hypotheses about a Population Mean

When testing hypotheses regarding a population mean, we use the ________________________________ (as opposed to the *z*-distribution), when the population standard deviation is unknown.

Since we don't know the population standard deviation, _________, we need to use the sample standard deviation, ______. As a result, the formula for the test statistic, ____, is shown below, and it follows **Student's *t*-distribution** with $n-1$ degrees of freedom.

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

Requirements Needed in Order to Use the P-value Method for Hypothesis Testing Regarding a Population Mean:

- Simple random sampling used to obtain the sample
- The sample contains no outliers and is drawn from a normally distributed population *OR* the sample size is large: ________________________
- The sample size is less than 5% of the population size ($n \leq 0.05N$) or sample values are independent of each other

Conducting a Hypothesis Test about a Population Mean:

Step 1: Determine and state the null and alternative hypotheses.

Remember, we have three options to set up the null and alternative hypotheses:

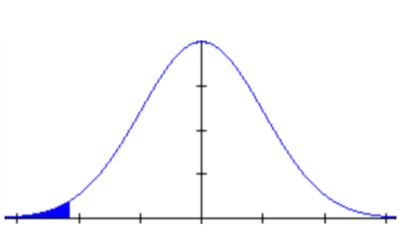
Left-Tailed	Two-Tailed	Right-Tailed
$H_0: \mu = \mu_0$	$H_0: \mu = \mu_0$	$H_0: \mu = \mu_0$
$H_1: \mu < \mu_0$	$H_1: \mu \neq \mu_0$	$H_1: \mu > \mu_0$

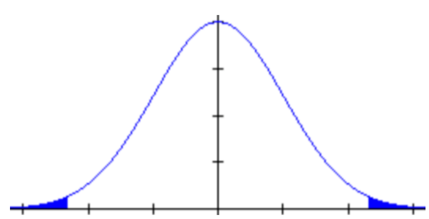
Step 2: Determine the level of significance, __________, typically given by the problem.

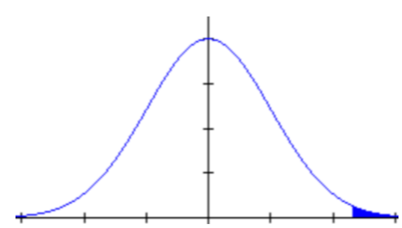
Step 3: Compute the test statistic: $t_0 = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$

Step 4: Find the corresponding P-value using the **tcdf** function. Remember, your instructor may require you to draw the curve.

Left-Tailed	Two-Tailed	Right-Tailed







***Note:** To use the **tcdf** function, input **tcdf**(*lower bound, upperbound, degrees of freedom)*

Step 5: Determine the Results of the Test then State the Conclusion

- If the ______________________, __________________ the null hypothesis

Example: Testing a Hypothesis about a Population Mean using a Large Sample

According to the American Screen-Watcher Watchers, the typical American spent 2.79 hours per day watching a screen of some sort back in 2015. Do Americans spend less time each day watching a screen now? 75 randomly selected Americans were observed (with their permission). The mean time watching a screen per day was 2.63 hours, with a standard deviation of 0.65 hours. Conduct the appropriate test to determine whether Americans spend less time each day watching a screen now than they did back in 2015. Use the $\alpha = 0.1$ level of significance.

First, we must **verify the requirements** to perform the hypothesis test:

- This is a simple random sample
- ________________________, so we have a large sample size
- Since the sample size is less than 5% of the population size, the assumption of independence is met

Hypothesis testing setup:

Step 1: H_0: _________________ and H_1: ____________________

Step 2: The level of significance is ________________

Step 3: Compute the test statistic:

$$t_0 = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

Step 4: Find the corresponding *P*-value using the **tcdf** function. Remember to draw the curve!

Step 5: Conclusion: There is _____________________evidence, at the $\alpha = 0.1$ level of significance, to conclude that mean time Americans spend watching a screen is ________________________ 2.79 hours per day.

Example: Testing a Hypothesis about a Population Mean using a Small Sample Size

The mean waiting time to get a table at California's best Haggis (dare you to look that up) restaurant is approximately 28.7 minutes. The owner of the restaurant feels so guilty that people have to wait so long to get their Haggis that he creates a new system to change the wait times. However, he's not exactly sure whether wait times will get better or worse. After implementing the system, he randomly selects 12 parties and measures the wait times, shown below. Test whether the new system changed the time parties had to wait to be seated at the α = 0.05 level of significance.

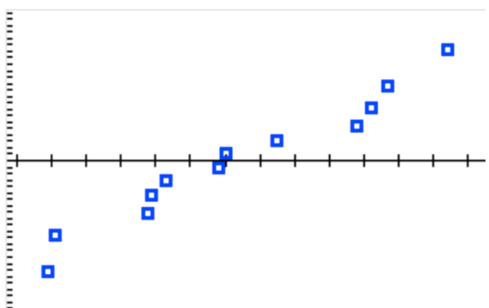
18.0	15.9	15.8	22.7
22.2	16.3	12.9	13.1
24.4	19.5	17.8	21.8

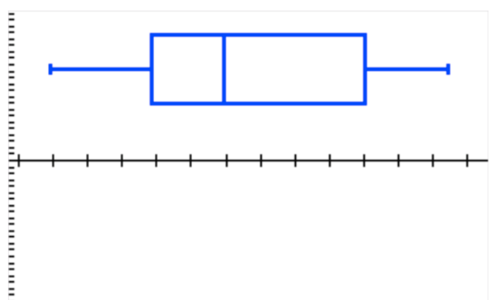
First, we must **verify the requirements** to perform the hypothesis test:

- This is a simple random sample
- ______________________________. Since the sample size is _______________, we must verify that the data come from a population that is __________________ distributed with no __________________ before proceeding
- Since the sample size is less than 5% of the population size, the assumption of independence is met

How do we verify that the data come from a population that is normally distributed with no outliers again?

Make a ***Normal Probability Plot*** and a ***Boxplot***





The normal probability plot of the data is approximately ____________________, and there are no ________________________ so we can proceed.

Hypothesis testing setup:

Step 1: H_0: __________________ and H_1: ______________________

Step 2: The level of significance is _________________

Step 3: Compute the test statistic:

$$t_0 = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

Step 4: Find the corresponding *P*-value using the **tcdf** function. Remember to draw the curve!

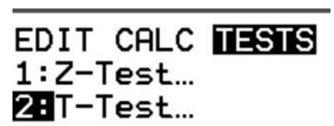
Step 5: Conclusion: There is ______________________evidence at the $\alpha = 0.05$ level of significance to conclude that the mean wait time is _____________________________28.7 minutes.

Final thought: You can CHECK your Hypothesis Test results using a calculator

The mean waiting time to get a table at California's best Haggis (dare you to look that up) restaurant is approximately 28.7 minutes. The owner of the restaurant feels so guilty that people have to wait so long to get their Haggis that he creates a new system to decrease the wait times. After implementing the system, he randomly selects 12 parties and measures the wait times, shown below. Test whether the new system decreased the time parties had to wait to be seated at the $\alpha = 0.05$ level of significance.

18.0	15.9	15.8	22.7
22.2	16.3	12.9	13.1
24.4	19.5	17.8	21.8

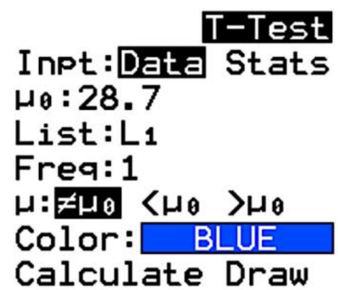
Input the given data into a list (I'm using L1) then **perform a T – test:**
Hit **Stat**, arrow to **Tests**, **Enter**, arrow to **T–test**, **Enter**



Input our given details **after hitting enter on Data**

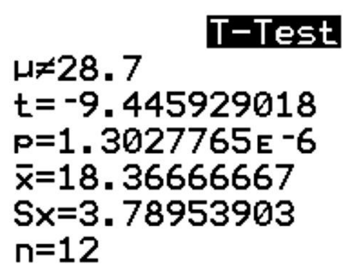
Inpt: Data (arrow to data and hit enter)
μ_0: 28.7
List: L1
Freq: 1
μ: $\neq \mu_0$

Then arrow to **Calculate** and hit **Enter**



The result we're looking for is *p*, which represents the *P*-value.

We get a *P*-value ≈ **0.000001**



Since the *P*-value is **less than** the level of significance (**0.000 < 0.05**) we **reject** the null hypothesis

Conclusion: There is **sufficient** evidence at the $\alpha = 0.05$ level of significance to conclude that the mean wait time has **changed**.

Pause the video, and try these problems. Resume the video to show the answers to each problem.

Example: The mean throwing distance of a football for a group of self-proclaimed “athletes” is 35 yards. A friend mocking the group says that they would be able to throw the ball farther if they had someone rushing at them to attempt a “sack” because adrenaline would kick in. The group attempts 40 more throws with the mocking friend bearing down on them, and the throws result in a mean distance of 38 yards and a standard deviation of 11 yards. Conduct a hypothesis test with $\alpha = 0.05$ to determine whether the group can throw footballs farther when someone is rushing at them.

Example: A college badminton team says the mean weight that they can bench press is 75 pounds (they’re super strong), with a standard deviation of 12 pounds. The team’s coach thinks that the mean weight is less than that amount. He randomly selected 14 of the players to lift on the bench press and recorded the maximum weights lifted (shown below). Conduct a hypothesis test at the $\alpha = 0.1$ level of significance to determine whether the mean weight was less than 75 pounds. Remember to confirm that the requirements of the test are met.

Results:
68 77 75 72 81 88 69
64 55 61 63 65 73 75

Objective 2 – Testing Hypotheses about a Population Standard Deviation

Requirements for Testing Hypotheses about a Population Variance or Standard Deviation:

- The sample is obtained using simple random sampling
- The population is normally distributed

Conducting a Hypothesis Test about Variance or Standard Deviation:

Step 1: Determine and state the null and alternative hypotheses. Remember, we have three options to set up the null and alternative hypotheses:

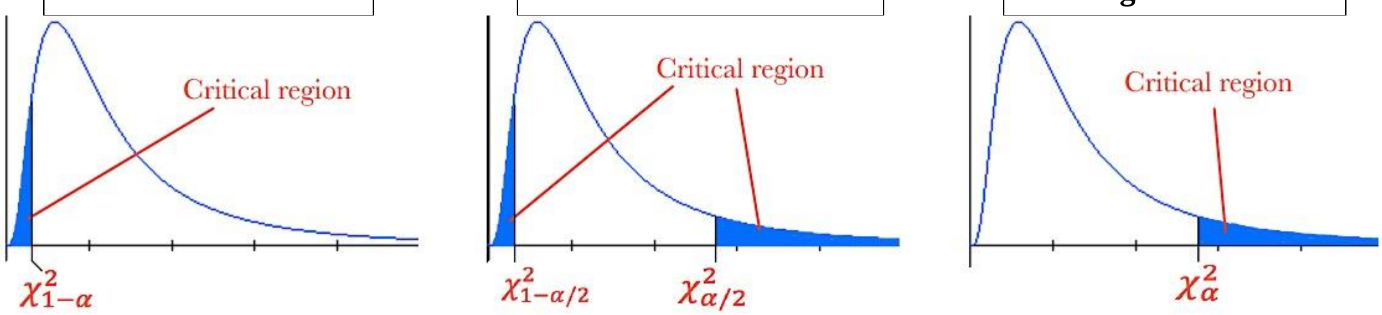
Left-Tailed	**Two-Tailed**	**Right-Tailed**
$H_0: \sigma = \sigma_0$	$H_0: \sigma = \sigma_0$	$H_0: \sigma = \sigma_0$
$H_1: \sigma < \sigma_0$	$H_1: \sigma \neq \sigma_0$	$H_1: \sigma > \sigma_0$

Step 2: Determine the level of significance, __________, typically given by the problem.

Step 3: Compute the test statistic: $\chi_0^2 = \frac{(n-1)s^2}{\sigma_0^2}$

and use the Chi-Square table to determine the critical value with *n* – 1 degrees of freedom.

Left-Tailed	**Two-Tailed**	**Right-Tailed**



In summary, **if the test statistic** falls in the __________________________________, we reject the null hypothesis.

Step 4: Compare the critical value with the test statistic:

Left-Tailed	**Two-Tailed**	**Right-Tailed**
If $\chi_0^2 < \chi^2_{1-\alpha}$,	If $\chi_0^2 < \chi^2_{1-\alpha/2}$ or $\chi_0^2 < \chi^2_{\alpha/2}$,	If $\chi_0^2 > \chi^2_{\alpha}$,
Reject the null hypothesis	Reject the null hypothesis	Reject the null hypothesis

Step 5: State the conclusion

Example: Testing a Hypothesis about a Population Standard Deviation

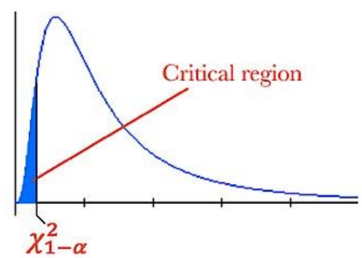
One measure of the risk of a being a responsible human being is the standard deviation of how much fun is lost as a direct result of being responsible. Suppose a mutual fund-like social group quantifies its "responsible decisions" as having moderate risk of losing life points if the standard deviation of its monthly rate of lost fun is less than 4%. The leader of this group claims that his fund has moderate risk, as he tries to sales pitch young people to start being more responsible, rather than 100% fun. A social group evaluation company (sure, that's a thing) randomly selects 25 months and determines the standard deviation of the rate of lost fun for the mutual fund-like group, which ends up being approximately 2.98%. Test the claim that the mutual fund-like group has a moderate risk at the α = 0.05 level of significance? Note that the monthly rates of lost fun are confirmed to be normally distributed.

Analysis: First, we must verify the requirements to perform the hypothesis test:

1) The sample is obtained using simple random sampling
2) The population is normally distributed

Hypothesis testing setup:

Step 1: H_0: __________ and H_1: ____________ So this test is ____________________



Step 2: The level of significance is _______________

Step 3: Calculate the χ_0^2 test statistic

$$\chi_0^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

Step 4: Compare the critical value with the test statistic:

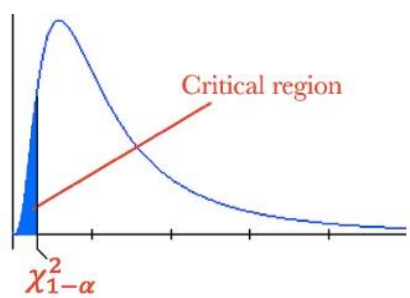
χ^2 distribution table

df	$\chi^2_{.995}$	$\chi^2_{.990}$	$\chi^2_{.975}$	$\chi^2_{.950}$	$\chi^2_{.900}$	$\chi^2_{.100}$	$\chi^2_{.050}$	$\chi^2_{.025}$	$\chi^2_{.010}$	$\chi^2_{.005}$
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928

df = ______________ $\chi^2_{1-\alpha} = \chi^2_{1-0.05} = \chi^2_{.95} =$ ____________________

We have the test statistic, $\chi_0^2 =$ ____________________,

and the critical value, $\chi^2_{1-\alpha} = \chi^2_{.95} =$ __________________.



Therefore, since χ_0^2 ______________ in the critical region, we __________________the null hypothesis.

Step 5: Conclusion: There is __________________evidence at the $\alpha = 0.05$ level of significance to conclude that the standard deviation is ________________________ 4%.

Pause the video, and try these problems. Resume the video to show the answers to each problem.

Example: A curious Kinesiology student thinks that people who cannot walk in a straight line (instead they zig-zag inexplicably) tend to incur more random, ridiculous injuries than "straight-line" walkers. In order to test this, the student conducts a hypothesis test about the standard deviation of people's steps, measuring the distance steps waiver (in inches) from a straight line. The student claims people who are injury prone have a higher standard deviation of wavering steps than 3.2 inches. He measures the standard deviation of a random sample of 10 people who he has deemed as injury prone. The distances wavering from a straight line have a standard deviation of 4.4 inches. Using an $\alpha = 0.05$ level of significance, conduct a hypothesis test to determine whether people who are more injury prone have a higher standard deviation in their wavering steps than 3.2 inches. Determine what parameter is being tested, state the null and alternative hypotheses, and state your conclusion of the test.