

Simplifying Advanced Trigonometric Expressions

Objective 1: Sum and Difference Identities

Along with the fundamental identities, you will also need to know more advanced identities to simplify trigonometric expressions.

Sum and Difference Identities

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Ex) Find the exact value of the expression (do not use a calculator).

a) $\sin \frac{\pi}{5} \cos \frac{7\pi}{15} + \cos \frac{\pi}{5} \sin \frac{7\pi}{15}$

b) $\cos \frac{5\pi}{8} \cos \frac{\pi}{8} + \sin \frac{5\pi}{8} \sin \frac{\pi}{8}$

c) $\frac{\tan 280^\circ - \tan 40^\circ}{1 + \tan 280^\circ \tan 40^\circ}$

Ex) Find the exact value of the given function without the use of a calculator.

a) $\sin\left(\frac{7\pi}{12}\right)$

b) $\tan\left(\frac{19\pi}{12}\right)$

c) $\cos(-75^\circ)$

d) $\tan\left(-\frac{5\pi}{12}\right)$

Ex) Simplify each expression as much as possible.

a) $\frac{\sin(\beta - \alpha)}{\sin \alpha \sin \beta}$

b) $\cos(\alpha + \beta) + \cos(\alpha - \beta)$

c) $\cos(x + y)\cos(x - y)$

Stop the video and try these examples. Re-start the video to check your solutions.

Extra Examples:

Find the exact value of the expression. (Do not use a calculator.)

1) $\frac{\tan 245^\circ + \tan 85^\circ}{1 - \tan 245^\circ \tan 85^\circ}$

2) $\sin 280^\circ \cos 55^\circ - \cos 280^\circ \sin 55^\circ$

Without the use of a calculator, find the exact value of the given function.

3) $\cot \frac{23\pi}{12}$

4) $\cos 255^\circ$

Simplify the expression as much as possible.

5) $\sin(x + y) + \sin(x - y)$

6) $\frac{\sin(\theta + \alpha)}{\sin \theta \sin \alpha}$

Objective 2: Double Angle Identities

Double Angle Identities

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta$$

$$= 2 \cos^2 \theta - 1$$

$$= 1 - 2 \sin^2 \theta$$

Ex) Find the exact value of the following expressions.

a) Find the exact value of $\tan 2\alpha$ if $\cos \alpha = -\frac{1}{5}$ where $\frac{\pi}{2} < \alpha < \pi$.

b) Find the exact value of $\sin 2\beta$ if $\cot \beta = \frac{2}{3}$ where $\pi < \beta < \frac{3\pi}{2}$.

Ex) Simplify each expression.

a) $(\sin x + \cos x)^2$

b) $2 \sin 5\theta - 4 \sin^3 5\theta \cos 5\theta$

c) $\frac{7 \tan 2t}{1 - \tan^2 2t}$

Stop the video and try these examples. Re-start the video to check your solutions.

Extra Examples:

1) Find the exact value of $\sin 2\theta$ if $\tan \theta = \frac{2}{3}$ where $\pi < \theta < \frac{3\pi}{2}$.

Simplify the expression as much as possible.

2) $\frac{\sin 2x}{\sin x} - \frac{\cos 2x}{\cos x}$

3) $\frac{\csc^2 \theta - 2}{\csc^2 \theta}$

Objective 3: Half Angle Identities and Power Reducing Identities

Half Angle Identities		Power Reducing Identities
$\sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}}$	$\tan \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}$ $= \frac{\sin \theta}{1 + \cos \theta}$ $= \frac{1 - \cos \theta}{\sin \theta}$	$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$
$\cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}}$		$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$

Ex) Find the exact value for each expression using a Half Angle Identity.

a) $\cos(105^\circ)$

b) $\tan \frac{7\pi}{8}$

Ex) Find each exact value.

a) Find $\cos \frac{x}{2}$ if $\frac{\pi}{2} < x < \pi$ and $\tan x = -4$.

b) Find $\tan \frac{\theta}{2}$ if $\frac{3\pi}{2} < \theta < 2\pi$ and $\csc \theta = -3$.

c) Find $\sin A$ if $\cos 2A = \frac{3}{4}$ and $\frac{3\pi}{2} < 2A < 2\pi$.

Ex) Simplify each expression.

a) $\cos x + 2\sin^2 \frac{x}{2}$

b) $2 - 2\cos^2(3\beta)$

Stop the video and try these examples. Re-start the video to check your solutions.

Extra Examples:

Calculate the exact value for the following expressions by using half angles.

1) $\tan 105^\circ$

2) $\cos \frac{11\pi}{12}$

Find the exact value.

3) Find the $\csc \frac{\theta}{2}$ if $\cot \theta = -\frac{5}{2}$ and $\frac{\pi}{2} < \theta < \pi$.

4) Find the $\tan \theta$ if $\sin 2\theta = -\frac{3}{4}$ and $\pi < 2\theta < \frac{3\pi}{2}$.

Simplify the expression as much as possible.

5) $\frac{1 - \cos 2\theta}{1 + \cos 2\theta}$