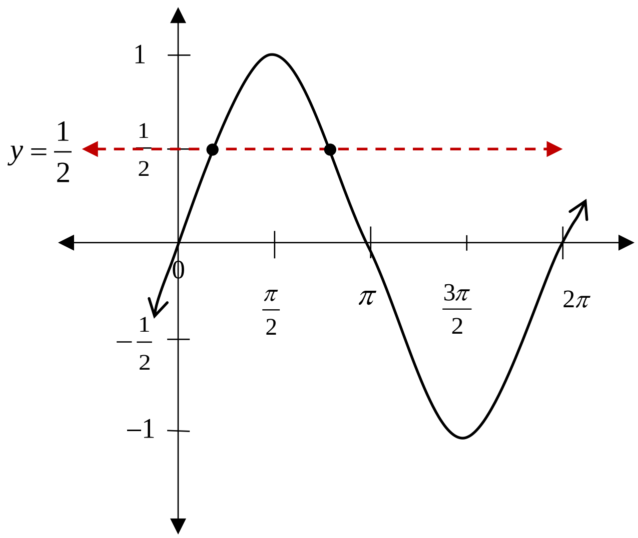
Solving Trigonometric Equations with Multiple Angles

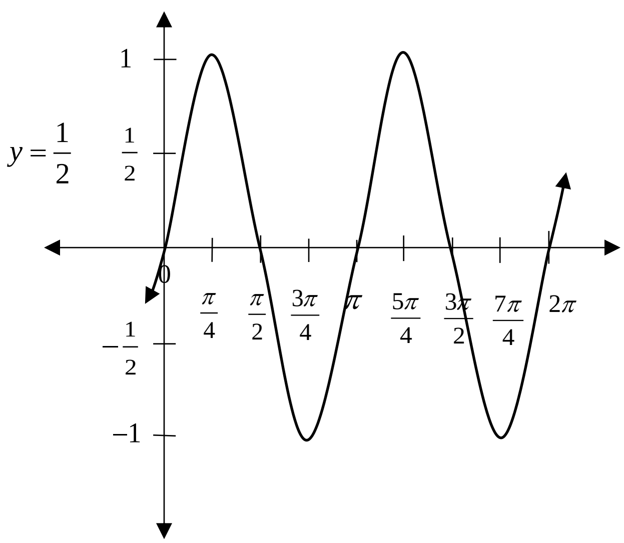
Objective 1: Graphic Introduction to Solving a Multiple Angle Trig Equation

Recall the graph of the equation $\sin x = \frac{1}{2}$ on the interval $[0, 2\pi)$.

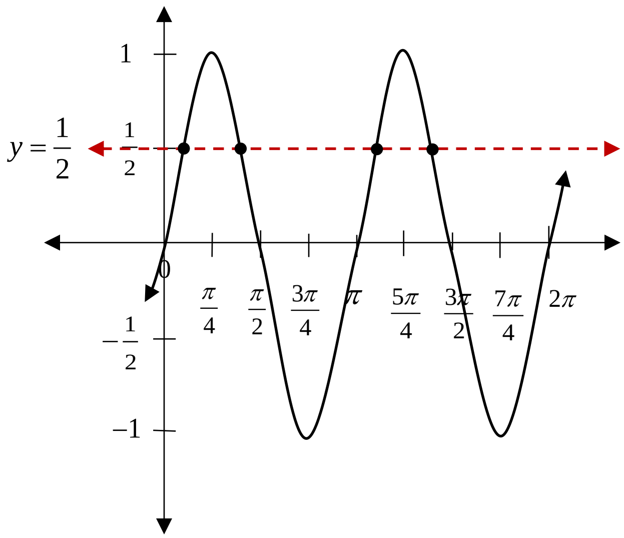


As you can see from the graph, we have only 2 solutions on the given interval. What happens when we have the equation $\sin(2x) = \frac{1}{2}$ on the given interval?

Start by graphing $y = \sin(2x)$ on the interval $[0, 2\pi)$



Then graph the line $y = \frac{1}{2}$ and notice how many points of intersection there are.

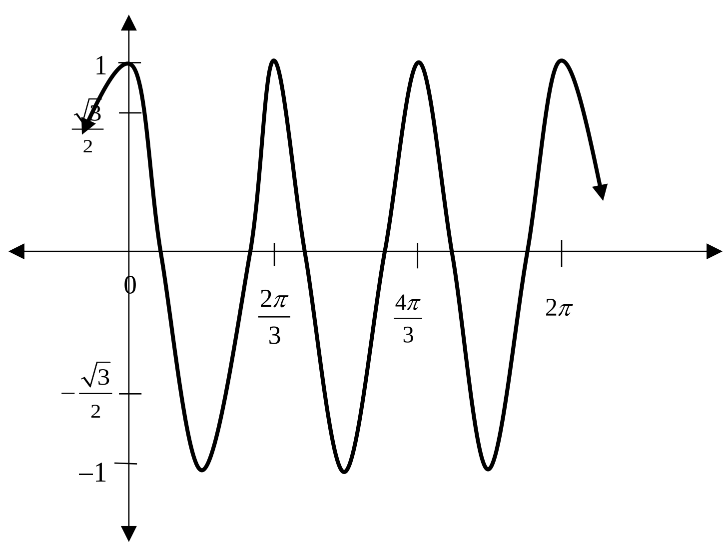


As you can see, there are now twice as many solutions than there were for the equation $\sin x = \frac{1}{2}$ on the interval $[0, 2\pi)$.

Demonstration:

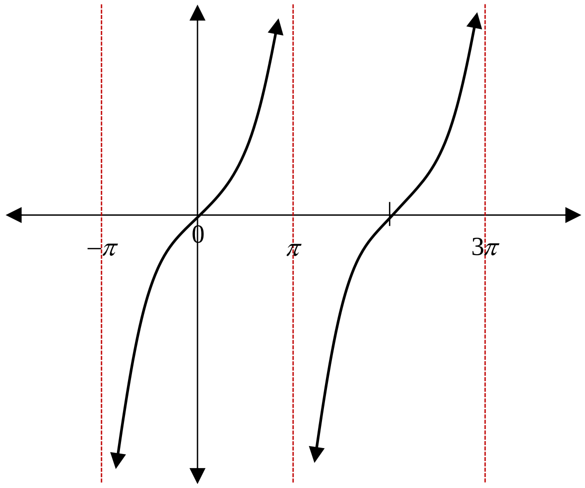
Given the graph of the function, state how many solutions there will be within the given interval.

a) $\cos(3x) = -\frac{\sqrt{3}}{2}$ over the interval $[0, 2\pi)$

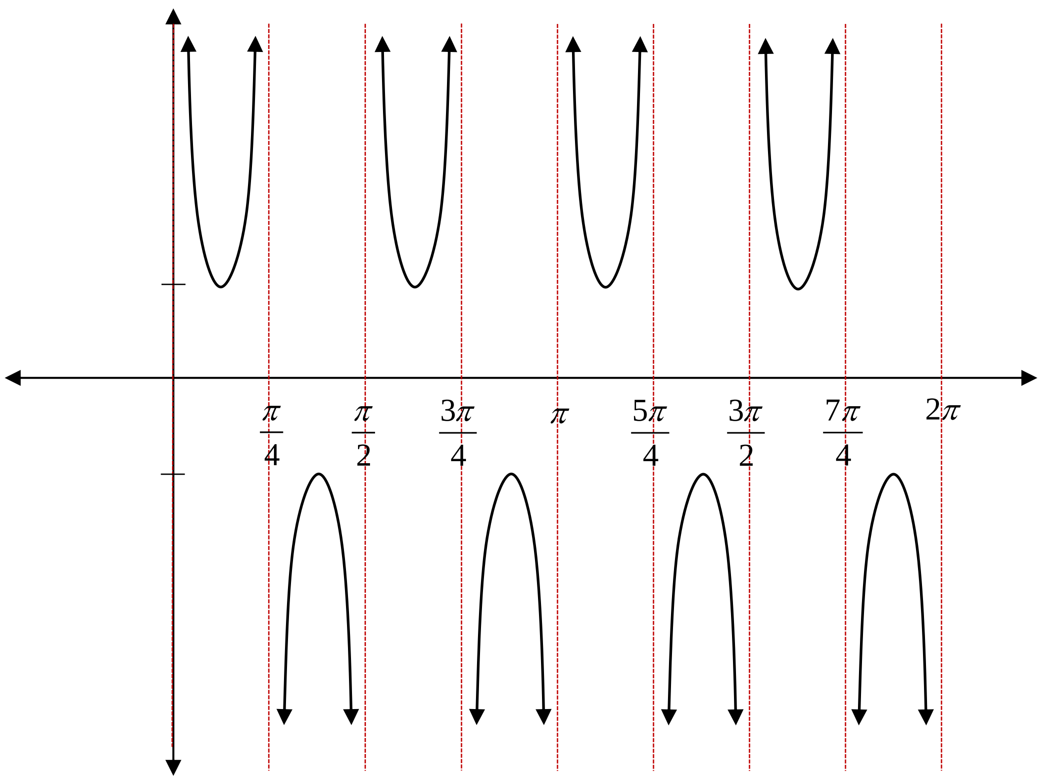


Given the graph of the function, state how many solutions there will be within the given interval.

b) $\tan\left(\frac{x}{2}\right)=\frac{\sqrt{2}}{2}$ over the interval $[0,2\pi)$



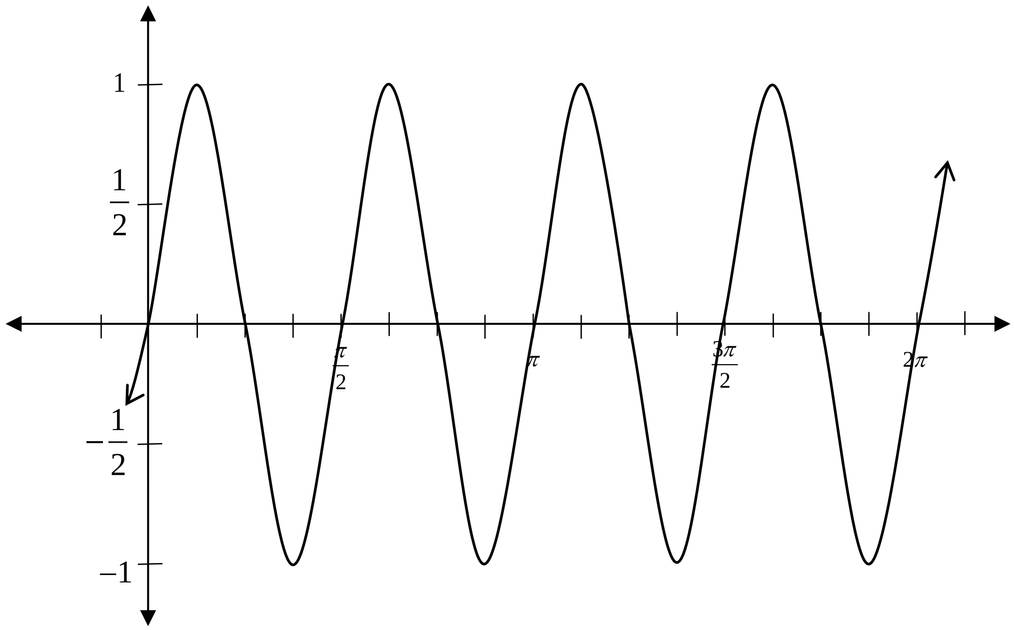
c) $\csc(4x)=1$ over the interval $[0,2\pi)$



Pause the video to try this one on your own, then restart when you are ready to check your answer.

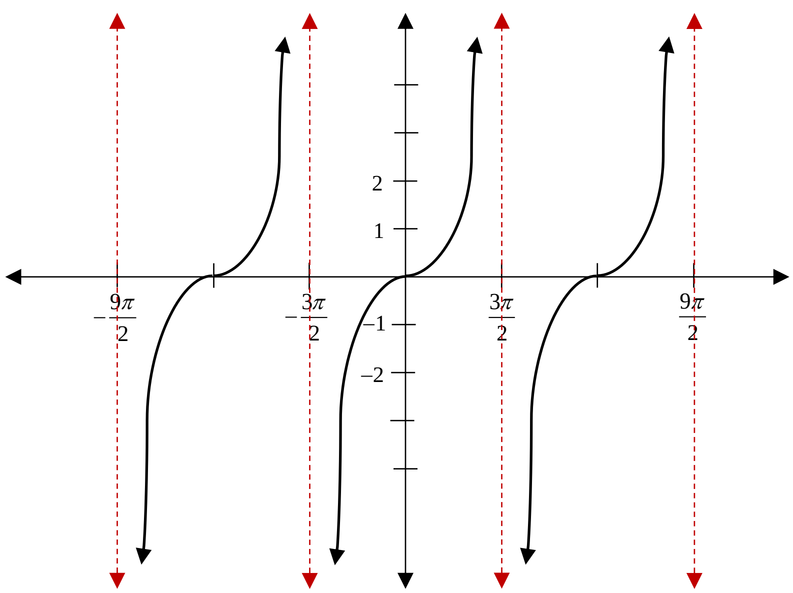
1) Given the graph of the function, state how many solutions there will be within the given interval.

$\sin(4x) = -\frac{1}{2}$ on the interval $[0, 2\pi)$



2) Given the graph of the function, state how many solutions there will be within the given interval.

$\tan\left(\frac{x}{3}\right) = -1$ on the interval $[-2\pi, 2\pi]$



Objective 2: Finding All Solutions

<u>Algebra Review</u>: **How to Clear a Fraction**

When solving an equation with fractions throughout the equation, you can opt to clear the equation of the fractions to help you during the process of solving.

Steps:

1) Multiply each term through the whole equation by the LCD.

2) If done correctly, every denominator should be cleared and the equation can be solved from there.

Ex) Solve:

$\frac{3}{4}x - \frac{1}{2} = \frac{5}{12}$ — 1) Multiply each term by LCD = 12

$(12)\frac{3}{4}x - (12)\frac{1}{2} = (12)\frac{5}{12}$ — 2) Cross cancel factors

$3(\cancel{12})\frac{3}{\cancel{4}}x - 6(\cancel{12})\frac{1}{\cancel{2}} = 1(\cancel{12})\frac{5}{\cancel{12}}$ — 3) Multiply the remaining factors and the numerator

$9x - 6 = 5$ — 4) Solve for the unknown value

$9x = 11$

$x = \frac{11}{9}$

Keep this in mind when solving a trig equation that involves multiple angles.

Steps to solving for ALL solutions involving multiple angles (General Solutions):

1) Solve the equation for the argument. Treat the argument as a single angle.
2) Find each angle that makes the statement true.
3) Set the argument equal to each angle and add either $2\pi n$ or $360°n$.
4) Finish solving for the variable by writing the solution as a single fraction. You can simplify each term as a single reduced fraction, but it will be more helpful to leave it as a single fraction.

Demonstration:

Ex) Solve for all exact solutions using radians.

$$2\cos\left(5x\right)-\sqrt{3}=0$$

Ex) Solve for all exact solutions using degrees.

$$4\csc\left(4\theta\right)-8=0$$

Ex) Solve for all exact solutions using radians.

$$5\tan\left(\frac{x}{2}\right)-3=2\tan\left(\frac{x}{2}\right)$$

Pause the video to try this one on your own, then restart when you are ready to check your answer.

Solve for all exact solutions in radians.

1) $\sqrt{3}\cot(5x)-3=-4$

Solve for all exact solutions in degrees.

2) $2\sin(4\theta)=\sqrt{2}+4\sin(4\theta)$

Solve for all exact solutions in radians.

3) $-8\cos\left(\frac{1}{2}x\right)-4=0$

Objective 3: Finding Particular Solutions on an Interval

Now that we have found all solutions, we can now use these solutions to find particular solutions on an interval.

Steps to finding particular solutions:

1) Find the general solutions of the equation. (See previous lesson)
2) Create a columned chart to help organize your solutions.
3) Start with $n=0$ and calculate the solutions until you reach the upper limit of the interval.

Demonstration

Ex) Find the exact solutions to the equation on the interval from $[0,2\pi)$

$$2\cos(3x)=-\sqrt{3}$$

$$\cos(3x)=-\frac{\sqrt{3}}{2}$$

$$(3x)=\frac{5\pi}{6} \text{ or } (3x)=\frac{7\pi}{6}$$

At this point, solve for ALL solutions in radians.

$$3x=\frac{5\pi}{6}+2\pi n \text{ or } 3x=\frac{7\pi}{6}+2\pi n$$

Clear the fraction and isolate the variable.

$$18x=5\pi+12\pi n \text{ or } 18x=7\pi+12\pi n$$

$$x=\frac{5\pi+12\pi n}{18} \text{ or } x=\frac{7\pi+12\pi n}{18}$$

Now we make a chart to help organize the multiple solutions we will have.

n	$x=\dfrac{5\pi+12\pi n}{18}$	$x=\dfrac{7\pi+12\pi n}{18}$

From here we will pick values of *n* starting at $n=0$, plugging it in until we reach the upper limit of the interval.

n	$x=\frac{5\pi+12\pi n}{18}$	$x=\frac{7\pi+12\pi n}{18}$
0	$x=\frac{5\pi+12\pi(0)}{18}=\frac{5\pi}{18}$	$x=\frac{7\pi+12\pi(0)}{18}=\frac{7\pi}{18}$
1	$x=\frac{5\pi+12\pi(1)}{18}=\frac{17\pi}{18}$	$x=\frac{7\pi+12\pi(1)}{18}=\frac{19\pi}{18}$
2	$x=\frac{5\pi+12\pi(2)}{18}=\frac{29\pi}{18}$	$x=\frac{7\pi+12\pi(2)}{18}=\frac{31\pi}{18}$
3	$x=\frac{5\pi+12\pi(3)}{18}=\frac{41\pi}{18}>2\pi$	

As you can see, when we plugged in $n=3$, we ended up with a solution greater than 2π, so we would stop at $\frac{31\pi}{18}$.

Ex) Solve for exact solutions on the interval $[0°,360°)$.

$$-2\sin 2\theta+1=0$$

Ex) Find the exact solutions to the equation on the interval from $[0, 2\pi)$.

$$\sqrt{3} + \cot 4x = 4\cot 4x$$

Ex) Solve for the unknown value in the interval $[0°, 360°)$. Round to the nearest tenth.

$$5\sec(3\theta) + 9 = 2\sec(3\theta) - 4$$

Ex) Solve for the equation using exact values over the interval $[0, 2\pi)$.

$$3\cos\left(\frac{x}{2}\right) = 2\sqrt{2} - \cos\left(\frac{x}{2}\right)$$

Pause the video to try this one on your own, then restart when you are ready to check your answer.

Solve for the exact solutions over $[0, 2\pi)$.

1) $\sin(4\theta) - 1 = -\sin(4\theta)$

Solve for the exact solutions over $[0°, 360°)$.

2) $4\tan(3x) = -4\sqrt{3}$

Solve for the exact solutions over $[0, 2\pi)$.

3) $3\cos(2x) = 2\sqrt{2} - \cos(2x)$

Using degrees, find the solutions to the equation over the interval $[0°, 360°)$. Round to the tenths.

4) $3\sec^2(3x) - 2\sec(3x) - 8 = 0$

Solve for the exact values over the interval $[0, 2\pi)$.

5) $2\sqrt{3}\sin\left(\frac{\theta}{2}\right) = 3$