

Cypress College Math Review:

Bernoulli Differential Equations

- A Bernoulli differential equation is one that can be written in the form $y' + p(x)y = q(x)y^n$ where n is any number other than 0 or 1. Write the equation in this form.
- Substituting $u = y^{1-n}$ makes the equation first-order linear.
- Find u' .
- Before making your substitution divide the equation by y^n , then substitute out all y 's.
- Obtain and simplify the integrating factor $\mu(x) = e^{\int p dx} = \exp(P(x))$, where $P(x)$ is any antiderivative of $p(x)$. Leave off the constant of integration and simplify.
- Solve for u by integrating $u \cdot \mu = \int (q \cdot \mu) dx$
- Substitute back and solve for y .

Example) $3xy^2y' - 3y^3 = x^4 \cos x$

Example) $2xy' + 3y + x^4y^3 = 0$

Example) $xy' - (3x + 6)y = -9xe^{-x}y^{4/3}$

Extra Practice – Try this problem on your own, then check with the answer below.

1. $2x^3y' - 3x^2y = y^3$

Answer

1. $y = \pm \sqrt{\frac{x^3}{C - x}}$, $y=0$ is also a solution