Binomial and Normal Distributions

Objective 1: Determining if an Experiment is a Binomial Experiment

For an experiment to be considered a binomial experiment, four things must hold:

1. The experiment is performed for a **fixed number of trials**. (We let n denote the number of trials.)
2. The trials are **independent**.
3. For each trial, there are only **two** mutually exclusive **outcomes** (success and failure).
4. The **probability of success is the same** for each trial. (We let p denote the probability of success.)

Example: Determine whether the following experiments are binomial experiments. Explain.

(a) According to a recent study, 33% of Americans, 23 years or older, have been arrested. A random sample of 500 Americans, 23 years or older, are asked whether or not they have been arrested.

(b) Mary is at the fair, playing pop the balloon with 6 darts. There are 20 balloons total, 15 which say "LOSE" and 5 which say "WIN".

(c) Bob flips a coin until the coin lands on heads.

Objective 2: Calculating Probabilities for a Binomial Distribution

You can use this table to help you calculate probabilities for binomial distributions.

Phrase	Math Symbol	Calculator	Example
exactly, equals, is	$P(X = x)$	$\text{binompdf}(n, p, x)$	"exactly 5…" $P(X = 5) = \text{binompdf}(n, p, 5)$
between a and b, inclusive	$P(a \leq X \leq b)$	$\text{binompdf}(n, p, a) + \cdots + \text{binompdf}(n, p, b)$ OR $\text{binomcdf}(n, p, b) - \text{binomcdf}(n, p, a - 1)$	"between 6 and 8, inclusive" $P(6 \leq X \leq 8) = \text{binompdf}(n, p, 6) + \text{binompdf}(n, p, 7) + \text{binompdf}(n, p, 8)$ OR "between 5 and 12, inclusive" $P(5 \leq X \leq 12) = \text{binomcdf}(n, p, 12) - \text{binomcdf}(n, p, 4)$
no more than, at most	$P(X \leq x)$	$\text{binomcdf}(n, p, x)$	"no more than 5" $P(X \leq 5) = \text{binomcdf}(n, p, 5)$
fewer than, less than	$P(X < x)$	$\text{binomcdf}(n, p, x - 1)$	"fewer than 5" $P(X < 5) = \text{binomcdf}(n, p, 4)$
at least, no less than	$P(X \geq x)$	$1 - \text{binomcdf}(n, p, x - 1)$	"at least 7" $P(X \geq 7) = 1 - \text{binomcdf}(n, p, 6)$
more than, greater than	$P(X > x)$	$1 - \text{binomcdf}(n, p, x)$	"more than 7" $P(X > 7) = 1 - \text{binomcdf}(n, p, 7)$
n = number of trials		**p = probability of success**	**x = number of success**

To get to either **binompdf** or **binomcdf** in your calculator, press 

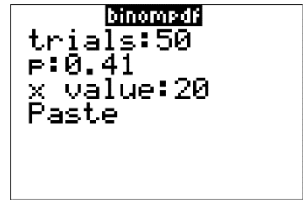
and scroll up until you find either **binompdf** or **binomcdf**.

Example: A study was done which stated that 41% of Americans only have a cell-phone in their house (no landline). What is the probability that in a random sample of 50 American households, that exactly 20 only have a cell-phone?

TI-83 Screen

```
binompdf(50,0.41,
20)
```

TI-84 Screen



Example: According to a recent article, 38% of buses in Chicago arrive on time. A random sample of 30 Chicago buses is taken.

(a) In a random sample of 30 Chicago buses, what is the probability that less than 10 arrive on time?

(b) In a random sample of 30 Chicago buses, what is the probability that exactly 17 arrive on time?

(c) In a random sample of 30 Chicago buses, what is the probability that at least 12 arrive on time?

(d) In a random sample of 30 Chicago buses, what is the probability that between 5 and 7, inclusive, arrive on time?

$$P(\text{between 5 and 7, inclusive}) =$$

$$P(\text{between 5 and 7, inclusive}) =$$

Objective 3: Finding Probabilities, Percents, or Proportions Using Normalcdf

Procedure: To find a probability, percent, or proportion for a normal distribution

Step 1: Draw the normal curve (optional).
Step 2: Calculate any z-scores using the formula $z = \frac{x-\mu}{\sigma}$.
Step 3: Find the probability using **normalcdf** and entering in the lower bound and upper bound.

(To get to **normalcdf** in your calculator press 

and select **normalcdf**.)

- If you have a TI-84, the following menu will appear. You will type your lower bound under lower and the upper bound under upper. Keep the mean, μ, at 0 and the standard deviation, σ, at 1.

Example: The weight of an American male is normally distributed with a mean of 199 pounds and a standard deviation of 15 pounds.

(a) What is the probability that a randomly selected American male will weigh less than 215 pounds?

Step 1 (optional) | *Step 2* | *Step 3*

Example (continued)**:** The weight of an American male is normally distributed with a mean of 199 pounds and a standard deviation of 15 pounds.

(b) What percent of American males weigh more than 185 pounds?

Step 1 (optional)	*Step 2*	*Step 3*

(c) What proportion of American males weigh between 150 and 175 pounds?

Step 1 (optional)	*Step 2*	*Step 3*

Objective 4: Finding the Value of a Normal Random Variable Using InvNorm

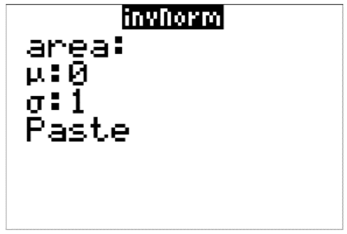
Procedure: To find a probability, percent, or proportion for a normal distribution

Step 1: Draw the normal curve (optional).

Step 2: Find any z-scores by using **invNorm** and entering in the area to the *LEFT* of the value you are trying to find. (To get to **invNorm** in your calculator press 

and select **invNorm**.)

- If you have a TI-84, the following menu will appear. You will type in the area to the left under area and keep the mean, μ, at 0 and the standard deviation, σ, at 1.



Step 3: Find the value of your random variable, x, by using the formula $x = \mu + z\sigma$.

Example: The weight of an American male is normally distributed with a mean of 199 pounds and a standard deviation of 15 pounds.

(a) Determine the 75th percentile for the weight of American males.

Step 1 (optional) | *Step 2* | *Step 3*

(b) Determine the weights that make up the middle 80% of weights for American males.

Step 1 (optional) | *Step 2* | *Step 3*