Domain and Range - Intermediate

Objective 1: Relations, Functions, Domain and Range

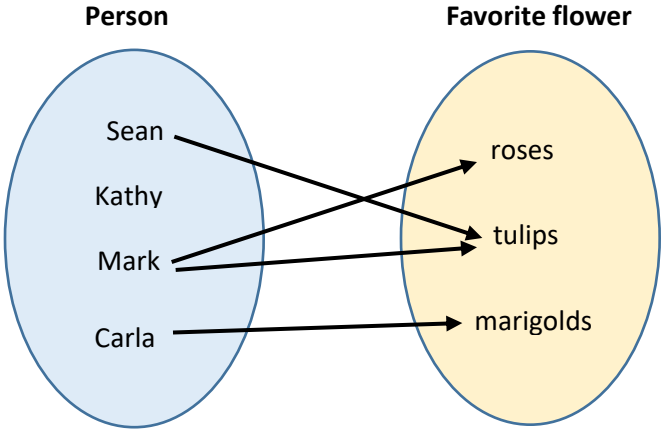
- A relation is a mapping from one set to another set.
- The first set is the domain.
- The second set is the range.
- A function is a special type of relation where each element in the domain is mapped to exactly one element in the range.

Example:

x	y
-2	3
0	6
1	-5
4	7

- -2 is mapped to 3, 0 is mapped to 6, 1 is mapped to -5 and 4 is mapped to 7.
- This relation is a function since each x value is mapped to exactly one y value.
- The domain is the set {-2,0,1,4} since these are all of the possible x values.
- The range is the set {-5,3,6,7} since these are all of the possible y values.

Example:



- This relation is not a function for two reasons
  -
  -
- Domain =
- Range =

Example:

{(-2,3), (0,2), (1,2)}

- _____
  - Note that there are two different x values that map to one y value. That is fine. You just can't have one x value map to more than one y value.
- Domain = _____
- Range = _____

Objective 1 Questions:

1. Determine whether the relation is a function.

Student	Height
Frank	5’ 9”
Loan	6’ 1”
Sung	5’ 9”

This relation maps a student to his height.

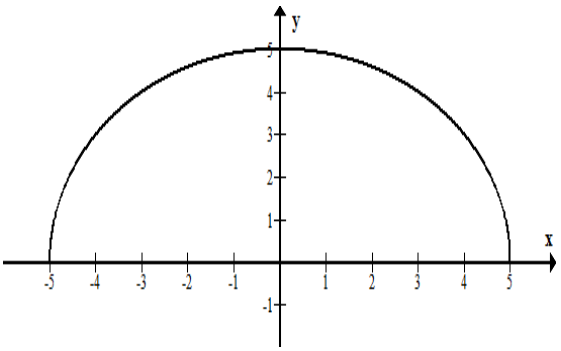
2. Determine the domain and range of the relation.

Student	Favorite Color
Mariana	Purple
Tenisha	Green
Brad	Yellow

This relation maps a person to their favorite color.

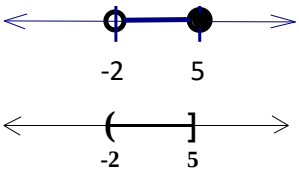
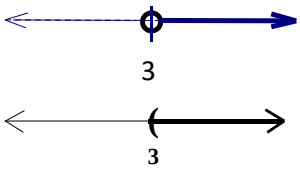
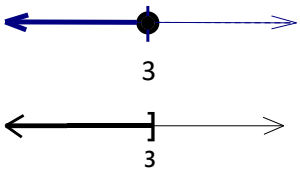
Objective 2: Determine the Domain and Range of a Relation from the Graph

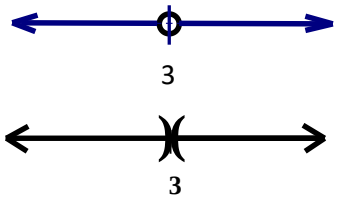
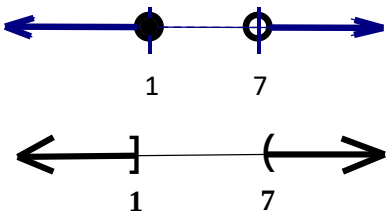
Example:



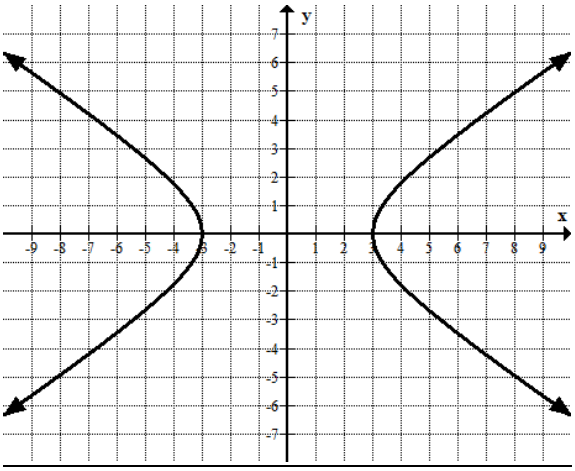
- This relation is a function, since each x value maps to exactly one y value. The way that you can tell by looking at the graph is if no matter where you draw a vertical line on the graph, you can NOT touch more than one of the points on the graph (vertical line test), then it is a function.
- Domain is [-5,5], which can also be written as $\{x \mid -5 \le x \le 5\}$ From the graph you look at how far left and how far right the graph goes, and if there are any breaks in the graph.
- Range is [0,5], which can also be written as $\{y \mid 0 \le y \le 5\}$ From the graph you look at how far down and how far up the graph goes, and if there are any breaks in the graph.

It is helpful to know both set builder notation and interval notation.

Graph			
Set builder notation			
Interval notation			

Graph		
Set builder notation		
Interval notation		

Example:

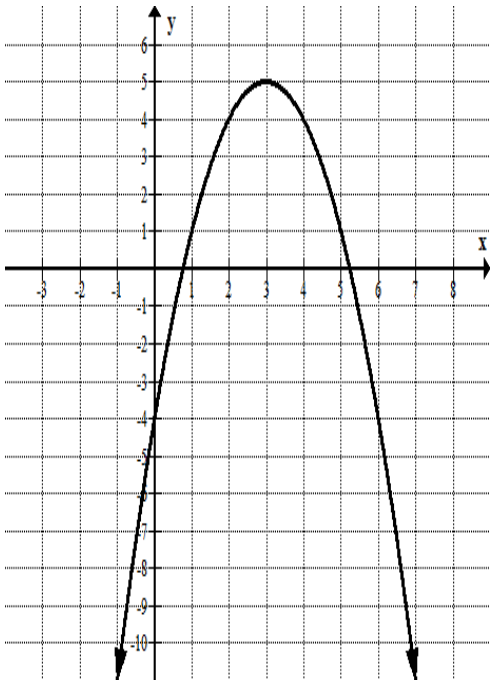


a) Is this relation a function? _____

b) Domain = _____

c) Range = _____

Example:



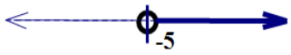
a) Is this relation a function? _____

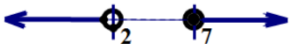
b) Domain = _____

c) Range = _____

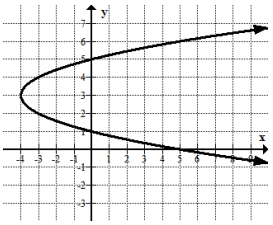
Objective 2 Questions:

Write the sets graphed in both set builder notation and interval notation.

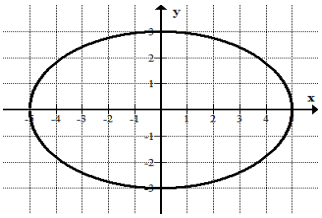
1. 

2. 

3. Determine by looking at the graph whether the relation is a function.



4. What are the domain and range of the given relation?



Objective 3: Determine the Domain of a Function from the Equation

$f(x)=\frac{4}{x}$ $g(x)=\frac{9}{x-3}$

What is f(4)? What is g(0)?

What is f(2)? What is g(3)?

What is f(0)?

If you are given an equation that defines a function and are not told any restrictions on the domain, then the domain is the set of all real numbers for which the expression is defined. So, we ask, when would it not be defined?

1. We know that we cannot divide by zero.
2. We know that we cannot take the square (or any even) root of a negative number.

These are the two restrictions we must look for. The domain is all OTHER real numbers.

Example: $f(x)=\dfrac{x+2}{x-3}$

1. Set the denominator equal to zero: x-3=0 so x=3
2. There are no square roots to worry about.

Domain
Set builder notation:

Interval notation:

Example: $f(x)=\sqrt{3x-2}$

1. There is no denominator to deal with.
2. What is inside the square root (called the radicand) must be ≥ 0. (recall $\sqrt{0}=0$ is fine)

Domain
Set builder notation:

Interval notation:

Example: $f(x)=2x^3-4x+7$

1. There is no denominator to deal with.
2. There are no square roots to worry about.

Domain

Example: $f(x)=\dfrac{\sqrt{x+1}}{x-3}$

1. Set the denominator equal to zero:
2. What is inside the square root (called the radicand) must be ≥ 0.

Domain
Set builder notation:
Interval notation:

Objective 3 Questions:

Find the domain of each function.

1. $f(x)=\sqrt{7+9x}$
2. $f(x)=\sqrt{4-x}$

3. $f(x)=\dfrac{x+6}{x-5}$

4. $f(x)=\dfrac{\sqrt{x+2}}{x-9}$

Objective 4: More Examples of Finding the Domain of Algebraic Functions

Example: $f(x)=\dfrac{\frac{5}{x}-7}{x+4}$

1. Set the denominator equal to zero.
2. What is **inside** the square root must be ≥ 0.

Domain
Set builder notation:

Interval notation:

Example: $f(x)=\dfrac{x+2}{\sqrt{3-x}}$

3. Set the denominator equal to zero.
4. What is **inside** the square root must be ≥ 0.

Domain
Set builder notation:

Interval notation:

Example: $f(x)=\sqrt{x^2+2x-15}$

1. There is no denominator to set equal to zero.
2. What is **inside** the square root must be ≥ 0.

Domain
Set builder notation:

Interval notation:

Example: $f(x)=\sqrt{x^2+3}$

1. There is no denominator to set equal to zero.
2. What is **inside** the square root must be ≥ 0.

Domain

Example: $f(x)=\sqrt{\dfrac{4x}{7-x}}$

1. Set the denominator equal to zero.
2. What is inside the square root must be ≥ 0.

Domain
Set builder notation:

Interval notation:

Objective 4 Questions:

Find the domain of each function.

1. $f(x)=\dfrac{x-\dfrac{3}{x+1}}{x-2}$

2. $f(x)=\dfrac{4x-3}{\sqrt{2x+1}}$

3. $f(x)=\sqrt{10+3x-x^2}$

4. $f(x)=\sqrt{\dfrac{x+2}{3x-4}}$

Objective 5: Domain of Exponential and Logarithmic Functions

Example: $f(x) = 5^x$

Domain =

Range =

Example: $f(x) = \log_5 x$

Domain =

Range =

Example: $f(x)=\log_7(x-3)$

Domain =

Example: $f(x)=\log_{\frac{1}{2}}(8-3x)$

Domain =

Example: $f(x)=\ln\left(\frac{x+3}{x-2}\right)$

Domain =

Objective 5 Questions:

Find the domain of each function.

1. $f(x) = 4^{x+3}$

2. $f(x) = \log_7(x-11)$

3. $f(x) = \ln(5-3x)$

4. $f(x) = \log_5\left(\frac{4+x}{1-x}\right)$