

Exponential and Logarithmic Equations

Objective 1: Solving Exponential Equations of the Form $a^m = a^n$

The Principle of Exponential Equality

If $a^m = a^n$ then $m = n$ for $a > 0$ and $a \neq -1, 0, 1$.

(Powers of the same base are equal if and only if the exponents are equal.)

Steps to Solving Exponential Equations

1. Use the Laws of Exponents to write both sides of the equation with the **same base**.
2. Set the exponential expressions on each side of the equation equal to each other.
3. Solve the equation resulting from Step 2.
4. Check the solution(s).

Example: Solve.

a. $6^x = 36$

b. $4^{3x-6} = 32^{2x}$

Objective 2: Definition of Logarithm

Definition of Logarithm

$$\log_a x = y \text{ means } a^y = x \text{ where } a > 0 \text{ and } a \neq 1$$

Note: $\log_{10} = \log$ (common log) and $\log_e = \ln$ (natural log)

Examples: Change each logarithmic expression to an equivalent expression involving an exponent.

a. $\log_a 4 = 5$

b. $\log_e b = -3$

c. $\log_3 5 = c$

Objective 3: Properties of Logarithm

Product Property of Logarithm

$$\log_a (XY) = \log_a X + \log_a Y \text{ for } a \neq 1$$

Examples: Write each expression as a single logarithm.

a. $\log_7 x + \log_7 4$

b. $\log 5 + \log 8$

Quotient Property of Logarithm

$$\log_a \frac{X}{Y} = \log_a X - \log_a Y$$

Examples: Write each expression as a single logarithm.

a. $\log_7 x - \log_7 4$

b. $\ln 5 - \ln 8$

Power Property of Logarithm

$$\log_a X^r = r \log_a X$$

Examples: Use the power property to rewrite each expression.

a. $\log_7 x^{-3}$

b. $\log_5 \sqrt[3]{x}$

Example: Write the expression as a sum and/or difference of logarithms. Express powers as factors.

$$\log_a \frac{8\sqrt{x}y^3}{z^5}$$

Write in expanded form:

- If it's in the **numerator**, it's **adding**.
- If it's in the **denominator**, it's **subtracting**.
- If it's an **exponent**, it will become a **factor**.

Example: Write the expression as a single logarithm.

$$\frac{1}{2} \log(x-2) - 5 \log y + 3 \log z$$

Write as a Single Logarithm:

- If it's a **factor**, it will become an **exponent**.
- If it's **adding**, it's in the **numerator**.
- If it's **subtracting**, it's in the **denominator**.

Objective 4: Solving Logarithmic Equations

Steps to Solving Logarithmic Equations

1. Combine to a single logarithm.
2. Remove log by applying the definition of logarithm.
3. Solve the equation.
4. Check the solution. If the value obtained in step 3 gives $\log(\text{a negative number})$, then disregard that value from the solution set.

Example: Solve the logarithmic equations. Express irrational solutions in exact form.

a. $\log_4(3x - 5) = 2$

b. $\log x + \log(x + 15) = 2$

Objective 5: Exponential Equations in General

Example: Solve the equation. Round the answer to four decimal places.

a. $8^{x-2} = 12$

b. $5 \cdot 2^{x+1} = 3$