

Exponentials and Logarithms - Basics

Objective 1: Find Values of Exponential Functions

A function, $f(x) = b^x$ where $b > 0$ and $b \neq 1$ is an exponential function.

$$f(x) = 3^x \qquad f(x) = 5^x \qquad f(x) = \left(\frac{1}{2}\right)^x$$

Example: $f(x) = 2^x$ is an exponential function.

- a. Find $f(-2)$
- b. Find $f(-1)$
- c. Find $f(0)$
- d. Find $f(1)$
- e. Find $f(2)$

Example: Complete the following t-chart for $f(x) = (5)^x$

x	y
-2	
-1	
0	
1	
2	

Example: Complete the following t-chart for $f(x) = \left(\frac{1}{3}\right)^x$

x	y
-2	
-1	
0	
1	
2	

Questions

1. Let $f(x) = 4^x$, find $f(-1)$
2. Let $f(x) = \left(\frac{1}{2}\right)^x$, find $f(-3)$
3. Let $f(x) = 5^x$, find $f(0)$
4. Let $f(x) = \left(\frac{1}{5}\right)^x$, find $f(2)$

Objective 2: Solve Exponential Equations with the Same Base

Example: If $3^4 = 3^x$ what is x ?

If $b > 0$ and $b \neq 1$, then

$b^x = b^y$ is equivalent to $x = y$.

Example: Solve: $2^{x+7} = 8$

Example: Solve: $9^{4x+2} = 27^x$

Example: Solve: $\frac{1}{125} = 25^{2x-3}$

Solve each of the following equations.

1. $64^x = 16$

2. $\frac{1}{27} = 9^{x-4}$

$$3. \left(\frac{1}{25}\right)^{x+2} = (125)^{3x}$$

Objective 3: Equivalent Exponential and Logarithmic Equations

If $b > 0$ and $b \neq 1$, then

$$y = \log_b x \text{ is equivalent to } b^y = x$$

Example: $2^3 = 8$

exponential form	logarithmic form

exponential form	logarithmic form

Common Logarithms
 $\log x$ means $\log_{10} x$

Natural Logarithm
 $\ln x$ means $\log_e x$

Questions. Write an equivalent exponential/logarithmic equation.

1. $3^4 = 81$

2. $\log_{16} 4 = \frac{1}{2}$

3. $7^{-2} = \frac{1}{49}$

4. $17^0 = 1$

5. $\log_{\frac{1}{4}} \left(\frac{1}{64} \right) = 3$

Objective 4: Find Values of Logarithmic Functions

Example: $\log_7 49$

Example: $\log_4 64$

Example: $\log_{36} 6$

Example: $\log_{\frac{1}{3}} 81$

Example: $\log_{15} 15$

Example: $\ln e^3$

Questions. Evaluate.

1. $\log_3 9$

2. $\log_7 \frac{1}{49}$

3. $\log_{49} 7$

4. $\log_{35} 1$

5. $\log_{\frac{1}{5}} 125$

Objective 5: Expanding and Contracting Logarithmic Expressions

$$\log_b(PQ) = \log_b P + \log_b Q$$

$$\log_b\left(\frac{P}{Q}\right) = \log_b P - \log_b Q$$

$$\log_b P^k = k(\log_b P)$$

Examples: Write as the sum and/or difference of logarithms. Express powers as factors.

$$\log_5(x^2\sqrt{y})$$

$$\log_5\frac{x}{y^4z^3}$$

Examples: Write as a single logarithm with a coefficient of one.

$$2 \cdot \log_7 x + \log_7 y$$

$$\log x - 4 \log y$$

Questions. Write as the sum and/or difference of logarithms. Express powers as factors.

1. $\log_5(x^4 y)$

2. $\ln \frac{x^2}{yz^3}$

Write as a single logarithm with a coefficient of one.

3. $3\log x - 2\log y$

4. $\frac{1}{2}(\log_7 x) + \log_7 y$

Objective 6: Solve Basic Logarithmic Equations

Example: Solve $\log_3(x + 7) = 2$

Example: Solve $\log(x + 7) - \log x = 2$

Example: Solve $\log_4 x + \log_4 (x + 6) = 2$

If $\log_b x = \log_b y$ then $x = y$

Example: Solve $\log 5 + \log (x - 3) = \log 4$

Example: Solve $\ln(2x + 1) + \ln(3) = \ln 15$

Questions. Solve.

1. $\log_5(2x - 1) = 1$

2. $\log_7 x + \log_7(x - 6) = 1$

3. $\log 3x - \log(x - 4) = 2$

4. $\ln(x + 4) + \ln 7 = \ln 9$