Exponentials and Logarithms - Intermediate

Objective 1: Find Values of Exponential Functions

A function, $f(x)=b^x$ where b>0 and b≠1 is an exponential function.

$f(x)=3^x$ $\quad$ $f(x)=5^x$ $\quad$ $f(x)=\left(\frac{1}{2}\right)^x$

Example: $f(x)=2^x$ is an exponential function.

a. Find $f(-2)$

b. Find $f(-1)$

c. Find $f(0)$

d. Find $f(1)$

e. Find $f(2)$

Example:

Complete the following t-chart for $f(x)=(5)^x$

x	y
-2	
-1	
0	
1	
2	

Example:

Complete the following t-chart for $f(x)=\left(\frac{1}{3}\right)^x$

x	y
-2	
-1	
0	
1	
2	

Questions

1. $Let\ f(x)=4^x,\ find\ f(-1)$

2. $Let\ f(x)=\left(\frac{1}{2}\right)^x,\ find\ f(-3)$

3. $Let\ f(x)=5^x,\ find\ f(0)$

4. $Let\ f(x)=\left(\frac{1}{5}\right)^x,\ find\ f(2)$

Objective 2: Solve Exponential Equations with the Same Base

Example: If $3^4=3^x$ what is x?

If b > 0 and b ≠ 1, then

$b^x=b^y\ is\ equivalent\ to\ x=y.$

Example: Solve: $2^{x+7}=8$

Example: Solve: $9^{4x+2}=27^x$

Example: Solve: $\frac{1}{125} = 25^{2x-3}$

Questions. Solve:

1. $64^{x} = 16$

2. $\frac{1}{27} = 9^{x-4}$

3. $\left(\frac{1}{25}\right)^{x+2} = (125)^{3x}$

Objective 3: Equivalent Exponential and Logarithmic Equations

If b > 0 and b ≠ 1, then

$y = \log_b x$ *is equivalent to* $b^y = x$

Example: $2^3 = 8$

exponential form	logarithmic form	exponential form	logarithmic form

Common Logarithms

$\log x$ *means* $\log_{10} x$

Natural Logarithm

$\ln x$ *means* $\log_e x$

Questions. Write an equivalent exponential/logarithmic equation.

1. $3^4 = 81$

2. $\log_{16} 4 = \frac{1}{2}$

3. $7^{-2} = \frac{1}{49}$

4. $17^0 = 1$

5. $\log_{1/4}\left(\frac{1}{64}\right)=3$

Objective 4: Find Values of Logarithmic Functions

Example: $\log_7 49$

Example: $\log_4 64$

Example: $\log_{36} 6$

Example: $\log_{1/3} 81$

Example: $\log_{15} 15$

Example: $\ln e^3$

Questions. Evaluate.

1. $\log_3 9$
2. $\log_7 \frac{1}{49}$
3. $\log_{49} 7$
4. $\log_{35} 1$
5. $\log_{1/5} 125$

Objective 5: Inverse Properties of Exponential and Logarithmic Functions

$$b^{\log_b x} = x$$

Example: $10^{\log 9}$

Example: $7^{\log_7 12}$

$$\log_b b^x = x$$

Example: $\log_{2/3}\left(\frac{4}{9}\right)$

Example: $\ln\sqrt{e}$

Questions. Find the exact value of each expression without using a calculator.

1. $\log_5 5^4$
2. $\ln\sqrt[3]{e}$
3. $\log_3\left(\frac{1}{81}\right)$
4. $5^{\log_5 7}$
5. $10^{\log 9}$

Objective 6: Expanding and Contracting Logarithmic Expressions

$$\log_b(PQ) = \log_b P + \log_b Q$$

$$\log_b\left(\frac{P}{Q}\right) = \log_b P - \log_b Q$$

$$\log_b P^k = k(\log_b P)$$

Examples: Write as the sum and/or difference of logarithms. Express powers as factors.

$\log_5\left(x^2\sqrt{y}\right)$ $\log_5 \frac{x}{y^4 z^3}$

Examples: Write as a single logarithm with a coefficient of one.

$2 \cdot \log_7 x + \log_7 y$ $\log x - 4\log y$

Questions. Write as the sum and/or difference of logarithms. Express powers as factors.

1. $\log_5\left(x^4 y\right)$

2. $\ln \frac{x^2}{yz^3}$

Write as a single logarithm with a coefficient of one.

3. $3\log x - 2\log y$

4. $\frac{1}{2}\left(\log_7 x\right) + \log_7 y$

Objective 7: Change Logarithmic Bases

Many calculators only have the capacity to approximate base 10 or base e logarithms. We need a method to convert other logarithms into these bases.

$$\log_b c = \frac{\log_a c}{\log_a b}$$

$$\log_b c = \frac{\log_{10} c}{\log_{10} b} = \frac{\log c}{\log b}$$

$$\log_b c = \frac{\log_e c}{\log_e b} = \frac{\ln c}{\ln b}$$

Example: Convert $\log_6 20$ to base 10 (common) logarithm. Use a calculator to approximate the value of the expression accurate to two decimal places.	**Example:** Convert $\log_6 20$ to base e (natural) logarithm. Use a calculator to approximate the value of the expression accurate to two decimal places.

Questions

1. Convert $\log_{3/5} 7$ to base 10 (common) logarithm. Use a calculator to approximate the value of the expression accurate to two decimal places.

2. Convert $\log_7 21$ to base e (natural) logarithm. Use a calculator to approximate the value of the expression accurate to two decimal places.

Objective 8: Solve Exponential Equations with Different Bases

If b > 0 and b $\neq$ 1, then

$$b^a = c \ \ is \ \ equivalent \ \ to \ \ a = \log_b c$$

Example: Solve $5^{7x-2} = 8$

Example: Solve $9e^{x+6} + 4 = 8$

Questions. Solve

1. $7^{5-x} = 9$

2. $6\left(2^{4x-5}\right)+1=3$

3. $8e^{x-3}-2=14$

Objective 9: Solve Basic Logarithmic Equations

Example: Solve $\log_3\left(x+7\right)=2$

Example: Solve $\log(x+7)-\log x=2$

Example: Solve $\log_4 x+\log_4(x+6)=2$

$$\boxed{\textit{If } \log_b x=\log_b y \textit{ then } x=y}$$

Example: Solve $\log 5+\log(x-3)=\log 4$

Example: Solve $\ln(2x+1)+\ln(3)=\ln 15$

Questions. Solve.

1. $\log_5(2x-1)=1$

2. $\log_7 x+\log_7(x-6)=1$

3. $\log 3x - \log(x-4) = 2$

4. $\ln(x+4) + \ln 7 = \ln 9$