

Cypress College Math Review:

First-Order Linear Differential Equations

Steps to solve a first-order linear differential equation:

1. Write it in standard form: $y' + p(x)y = q(x)$.
2. Obtain and simplify the integrating factor $\mu(x) = e^{\int p dx} = \exp(P(x))$,
where $P(x)$ is any antiderivative of $p(x)$. Leave off the constant of integration and simplify.
3. The solution is obtained by integrating $y \cdot \mu = \int (q \cdot \mu) dx$

Example) $(3x - 1)y' - 6y = -10(3x - 1)^{1/3}$

Example) $(1 + x^2)y' - 2xy = x^4 + x^2$

Example) $y' + \tan x y = \cos x$

Example) $y' + \frac{3x+1}{x}y = \frac{e^{-3x}}{x}$

Extra Practice – Try these on your own, then check with the answers below.

1. $y' + \cot x y = 3 \sin x \cos x$

2. $y' + \tan x y = \sec x$

Answers

1. $y = \sin^2 x + C_1 \csc x$

2. $y = \sin x + C_1 \cos x$