Integration Using Trigonometric Substitution

This method is useful when the integrals contain: $\sqrt{a^2-u^2}$, $\sqrt{a^2+u^2}$ or $\sqrt{u^2-a^2}$.

It provides a way to eliminate the radical.

- For integrals involving $\sqrt{a^2-u^2}$, let u = $a\sin\theta$
- For integrals involving $\sqrt{a^2+u^2}$, let u = $a\tan\theta$
- For integrals involving $\sqrt{u^2-a^2}$, let u = $a\sec\theta$

Find the integral.

1. $\displaystyle\int \frac{1}{x^2\sqrt{9-x^2}}\,dx$

Remark: Be careful to correctly substitute for dx.

2. $\int \frac{1}{\sqrt{4+9x^2}}\,dx$

3. $\int \frac{\sqrt{x^2-3}}{x}\,dx$

4. $\displaystyle\int \frac{-e^x}{(e^{2x}-1)^{3/2}}\,dx$

PRACTICE PROBLEMS

Find the integral.

1. $\int \frac{1}{\sqrt{x^2-25}}\,dx$

2. $\int \frac{1}{\sqrt{16-x^2}}\,dx$

3. $\int \frac{1}{x\sqrt{4x^2+9}}\,dx$

4. $\int \frac{-3x}{(x^2+3)^{3/2}}\,dx$

Restart the video when you are ready to check your answers.

Answers:

1. $\ln\left|x+\sqrt{x^2-25}\right|+C$
2. $\arcsin\left(\frac{x}{4}\right)+C$
3. $-\frac{1}{3}\ln\left|\frac{\sqrt{4x^2+9}+3}{2x}\right|+C$
4. $\frac{3}{\sqrt{x^2+3}}+C$