Cypress College Math Review: Linear Transformations

A transformation, T, is linear if:

for every scalar c and $\vec{u}, \vec{v} \in V$

1. $T(\vec{u}+\vec{v}) = T(\vec{u}) + T(\vec{v})$ these "+" may mean different things
2. $T(c\vec{u}) = cT(\vec{u})$

Determine whether the following transformations are linear

Example) $T:\mathbb{R}^2 \to \mathbb{R}^2$ $T(x_1,x_2) = (4x_1 - x_2, x_1 + 2x_2)$

Example) $T: M_{2\times 2} \to \mathbf{P}^2 \quad T\begin{bmatrix} a & b \\ c & d \end{bmatrix} = (a+b)x^2 - cx + d$

(where $\mathbf{P}^2$ is the set of all polynomials of degree less than or equal to 2)

Example) $T:\mathbf{P}^2 \to \mathbb{R}^3 \; T\left(ax^2+bx+c\right)=\left\langle 3b+c, a-c, 7\right\rangle$

Example) $T:\mathbf{P}^3 \to \mathbf{P}^3 \;\; Tf(x) = xf'(x)$

The Kernel (or nullspace) of a transformation is the set of all vectors $\vec{v} \in V \ \ where\ T\vec{v} = \vec{0}$

Example) Find the kernel of $T : \mathbf{P}^2 \to \mathbf{P}^1$, where $T\left\langle x_1, x_2 \right\rangle = \left\langle 4x_1 - x_2, x_1 + 2x_2 \right\rangle$

Example) Find the kernel of $T : \mathbb{R}^2 \to \mathbb{R}^2$, where $T\left(ax^2 + bx + c\right) = 2ax + b$

Extra Practice – Try these on your own, then check with the answers on the video.

1. $\vec{u} = \langle 1, 2, -3 \rangle$, $\vec{v} = \langle -4, 3, 5 \rangle$, $\vec{w} = \langle -10, 2, 16 \rangle$

2. $\vec{v_1} = \begin{bmatrix} x^2 - 3 \\ 4x \\ x + 2 \end{bmatrix}$, $\vec{v_2} = \begin{bmatrix} 5x - 1 \\ 3x^2 + 2 \\ 4 \end{bmatrix}$,