Cypress College Math Review: Linear Independence

A finite set $\left\{\overrightarrow{v_1}, \overrightarrow{v_2}, \overrightarrow{v_3}, \ldots, \overrightarrow{v_n}\right\}$ of elements of V is **linearly dependent** if there exist numbers c_1, c_2, c_3, . . . , c_n <u>not all zero</u> such that $c_1\overrightarrow{v_1} + c_2\overrightarrow{v_2} + c_3\overrightarrow{v_3} + \ldots + c_n\overrightarrow{v_n} = \vec{0}$

Let $c_i \neq 0$ then $\frac{c_1}{c_i}\overrightarrow{v_1} + \frac{c_2}{c_i}\overrightarrow{v_2} + \frac{c_3}{c_i}\overrightarrow{v_3} + \ldots + \frac{c_n}{c_i}\overrightarrow{v_n} = \overrightarrow{v_i}$ where i is missing in the indices on the left.

Hence, a set of vectors is **<u>linearly dependent</u>** if **one of the vectors can be written as a linear combination of the other vectors.**

$\overrightarrow{v_i} = c_1\overrightarrow{v_1} + c_2\overrightarrow{v_2} + c_3\overrightarrow{v_3} + \ldots + c_n\overrightarrow{v_n}$ for some i.

If the vector equation $c_1\overrightarrow{v_1} + c_2\overrightarrow{v_2} + c_3\overrightarrow{v_3} + \ldots + c_n\overrightarrow{v_n} = \vec{0}$ has only the trivial solution, then the set of vectors $\left\{\overrightarrow{v_1}, \overrightarrow{v_2}, \overrightarrow{v_3}, \ldots, \overrightarrow{v_n}\right\}$ is linearly independent.

If the number of vectors is larger than the dimension of the vector space, then the set of vectors must be linearly dependent.

Prove that the following are Linearly Independent or Linearly Dependent.

Example) $\vec{u} = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 9 & 7 \\ 6 & -2 \end{bmatrix}$, $\vec{w} = \begin{bmatrix} 0 & -3 \\ 1 & 5 \end{bmatrix}$

Example) $\vec{u} = x^2 + 3, \vec{v} = 4x - 1$

Example) $\vec{u} = \langle 3,6 \rangle$, $\vec{v} = \langle -2,-4 \rangle$

Extra Practice – Try these on your own, then check with the answers below.

Prove that the following are Linearly Independent or Linearly Dependent.

1. $\vec{u} = \langle 0,1,3 \rangle$, $\vec{v} = \langle -6,7,4 \rangle$, $\vec{w} = \langle 4,3,1 \rangle$
2. $\overrightarrow{v_1} = 4x - 5$, $\overrightarrow{v_2} = x^2 - 3x$
3. $\vec{u} = \begin{bmatrix} 1 & 3 \\ 2 & -5 \end{bmatrix}$, $\vec{v} = \begin{bmatrix} 3 & 9 \\ 6 & -15 \end{bmatrix}$

Answers

1. Linearly Independent
2. Linearly Independent
3. Linearly Dependent