

Cypress College Math Review: Partial Fraction Decomposition

- Partial fraction decomposition is a method of breaking up a complicated rational expression into the sum of simpler ones.
- First factor the denominator of the rational expression.
- Based on the chart below determine what terms will be in your decomposition.

Factor in denominator:	Term(s) in decomposition:
$ax + b$ first power of a linear	$\frac{A}{ax + b}$
$(ax + b)^m$ mth power of a linear	$\frac{A_1}{ax + b} + \frac{A_2}{(ax + b)^2} + \frac{A_3}{(ax + b)^3} + \dots + \frac{A_m}{(ax + b)^m}$
$ax^2 + bx + c$ irreducible quadratic (has no real zeros)	$\frac{Ax + B}{ax^2 + bx + c}$
$(ax^2 + bx + c)^m$ mth power of an irreducible quadratic	$\frac{A_1x + B_1}{ax^2 + bx + c} + \frac{A_2x + B_2}{(ax^2 + bx + c)^2} + \frac{A_3x + B_3}{(ax^2 + bx + c)^3} + \dots + \frac{A_mx + B_m}{(ax^2 + bx + c)^m}$

Example: $\frac{3x}{x^2 + x - 2}$

- Before breaking up a rational expression using partial fraction decomposition you must first make sure that the expression is proper.
- A proper rational expression is one in which the degree of the numerator is less than the degree of the denominator.
- If the rational expression is improper, then divide first. Use partial fraction decomposition to break apart the rational expression that you are left with.

Example: $\frac{x^3 + x^2 - 3}{x^2 + 3x - 4}$

Example: $\frac{x^2 - 4x + 7}{(x + 1)(x^2 - 2x + 3)}$

Example: $\frac{x - 3}{(x + 2)(x + 1)^2}$

Extra Practice – Try these on your own, then check with the answers below.

1. $\frac{5}{x^2 + x - 6}$

2. $\frac{x - 6}{2x^3 + 8x}$

3. $\frac{x^3 - x + 3}{x^2 + x - 2}$

4. $\frac{2x - 3}{(x - 1)^2}$

Answers

1. $\frac{-1}{x + 3} + \frac{1}{x - 2}$

2. $\frac{-3}{4x} + \frac{3x + 2}{4(x^2 + 4)}$ or $\frac{-\frac{3}{4}x + \frac{1}{2}}{x^2 + 4} - \frac{3}{4x}$

3. $x - 1 + \frac{1}{x + 2} + \frac{1}{x - 1}$

4. $\frac{2}{x - 1} - \frac{1}{(x - 1)^2}$