

Solving Linear Equations with Fractions and Answers as “No Solution” or “All Real Numbers”

Objective 1: Solving Linear Equations Involving Fractions

Example: Solve for x in the equation $\frac{2}{5}x = 10$.

Solution:

Method 1	Method 2
$\frac{2}{5}x = 10$	$\frac{2}{5}x = 10$
$5 \cdot \frac{2}{5}x = 10 \cdot 5$	Multiply by the reciprocal $\frac{5}{2} \cdot \frac{2}{5}x = 10 \cdot \frac{5}{2}$
$2x = 50$	$x = \frac{50}{2}$
$\frac{2x}{2} = \frac{50}{2}$	$x = 25$
$x = 25$	

Example: Solve for x .

$$\frac{x-1}{3} + \frac{2}{3} = x - \frac{2x+3}{9}$$

Solution:

Multiply each term by the LCD 9 to clear the fractions

$$9 \cdot \frac{x-1}{3} + 9 \cdot \frac{2}{3} = 9 \cdot x - 9 \cdot \frac{2x+3}{9}$$

Cross cancel the LCD with the denominators

$$3(x-1) + 3(2) = 9x - 1(2x+3)$$

Distribute

$$3x - 3 + 6 = 9x - 2x - 3$$

Combine like terms

$$3x + 3 = 7x - 3$$

Gather the terms involving x to the left

$$\begin{array}{r} -7x \quad -7x \\ \hline -4x + 3 = -3 \end{array}$$

Divide by -4 to get x alone

$$\begin{array}{r} -3 \quad -3 \\ \hline -4x = -6 \end{array}$$

Reduce

$$x = \frac{-6}{-4}$$

$$x = \frac{3}{2}$$

Example: Solve for x .

$$\frac{1}{10}(x + 1) = \frac{1}{6}(2 - x)$$

Solution:

$$30 \cdot \frac{1}{10}(x + 1) = 30 \cdot \frac{1}{6}(2 - x)$$

$$3(x + 1) = 5(2 - x)$$

$$3x + 3 = 10 - 5x$$

$$\begin{array}{r} +5x \qquad \qquad +5x \\ \hline \end{array}$$

$$8x + 3 = 10$$

$$\begin{array}{r} -3 \quad -3 \\ \hline \end{array}$$

$$8x = 7$$

$$x = \frac{7}{8}$$

Objective 2: Solving Linear Equations with Answers as “No Solution” or “All real numbers”

Example: Solve for x .

$$5x - 1 = 5(x + 3)$$

Solution:

$$5x - 1 = 5(x + 3)$$

$$5x - 1 = 5x + 15$$

$$\begin{array}{r} -5x \qquad \qquad -5x \\ \hline \end{array}$$

$$-1 = 15$$

This is a false statement because -1 does not equal to 15 . Therefore, the answer is “no solution.”

<p>Answer: No solution</p>

Example: Solve for x .

$$3x + 2(x + 4) = 5(x + 1) + 3$$

Solution:

$$3x + 2(x + 4) = 5(x + 1) + 3$$

$$3x + 2x + 8 = 5x + 5 + 3$$

$$5x + 8 = 5x + 8$$

$$\underline{-5x} \quad \underline{-5x}$$

$$8 = 8$$

This is a true statement because 8 does equal to 8. Therefore, the answer is “all real numbers.”

Answer:

All real numbers