Solving Radical Equations – Intermediate

Recall:

$\sqrt[n]{a}$ is read "the n^{th} root of a", where n is called the **index** and a is the **radicand**.

A **radical equation** is an equation in which the variable is in the radicand of a radical.

Examples of radical equations: $\sqrt{2x+1}=7,\ \sqrt[3]{x-5}=4,\ \sqrt[4]{x+2}=3$

Objective 1: Solving Radical Equations with Square Root

Steps to Solving a Radical Equation with Square Root:

1. Isolate the radical on one side of the equation.
2. Square both sides of the equation.
3. Solve the new equation.
4. Check the answer.

Example: Solve: $\sqrt{2x+1}=7$.

Example: Solve: $\sqrt{3x-2}=5$.

Example: Solve: $\sqrt{5x-4}\ -9=0$.

Example: Solve: $\sqrt{2x-1}+5=9$.

Example: Solve: $\sqrt{3x+9}-2=5$.

Example: Solve: $2\sqrt{4x+7}\ -3=9$.

Example: Solve: $5\sqrt{x-8}+2=10$.

Solve the following equations.

1. $\sqrt{4x+8} = 6$

2. $\sqrt{3x-5} = 5$

3. $\sqrt{3x+2} - 5 = 0$

4. $\sqrt{5x-1} + 4 = 6$

5. $\sqrt{9x+5} - 3 = 4$

6. $3\sqrt{2x-9} + 2 = 8$

7. $4\sqrt{3x+2} + 5 = 16$

Objective 2: Radical Equations with No Solution

If an equation has a square root equal to a negative number, that equation will have no solution.

Example: Solve: $\sqrt{9x-5} + 4 = 0$.

Example: Solve: $\sqrt{2x+8}+7=3$.

Solve the following equations.

1. $\sqrt{x+4}+6=0$

2. $\sqrt{6x-3}+12=5$

3. $\sqrt{2x+3}+8=4$

Objective 3: Solving Radical Equations with Higher Roots

Steps to Solving a Radical Equation with Higher Root:

1. Isolate the radical on one side of the equation.
2. Raise both sides of the equation to the power of the index.
3. Solve the new equation.
4. Check the answer.

Example: Solve: $\sqrt[3]{5x-1}+4=8$.

Example: Solve: $\sqrt[3]{2x-10}+1=-3$.

Example: Solve: $\sqrt[4]{3x-2}+3=5$.

Solve the following equations.

1. $\sqrt[3]{4x-8}+3=5$

2. $\sqrt[3]{x+10}+8=5$

3. $\sqrt[4]{2x+9}-1=2$

4. $\sqrt[5]{6x-1} = 2$

Objective 4: Radical Equations Having Two Solutions or Extraneous Solution

Remark: An extraneous solution is an extra solution that comes from the correct steps but does not actually work in the original equation.

We still follow the same four steps to solve a radical equation as those in Objective 1.

Example: Solve: $\sqrt{w-1} + 1 = w$

Example: Solve: $\sqrt{10-x} + x = 8$

Example: Solve: $2 = 2\sqrt{x + 17} - x$

Example: Solve: $\sqrt{-4y + 17} = y - 3$

Solve the following equations.

1. $\sqrt{x-2}+2=x$

2. $\sqrt{x+1}-x+1=0$

3. $\sqrt{x+9}-x+3=0$

4. $x + 1 = \sqrt{5x - 1}$

Objective 5: Radical Equations Containing Two Radicals

Steps to Solving a Radical Equation Containing Two Radicals:

1. Isolate a radical on one side of the equation.
2. Raise both sides of the equation to the power of the index.
3. Are there any more radicals?
   If yes, repeat step 1 and step 2 again.
   If no, go to step 4.
4. Solve the new equation.
5. Check the answer.

Example: Solve: $\sqrt{3m + 7} = \sqrt{m^2 + 3}$

Example: Solve: $2\sqrt{3x+5} = \sqrt{6x+44}$

Example: Solve: $\sqrt{x} + 3 = \sqrt{x+12}$

Example: Solve: $\sqrt{x+5}+4=\sqrt{x+85}$

Example: Solve: $\sqrt{14x-55}=\sqrt{13x-64}+1$

Solve the following equations.

1. $\sqrt{k^2+8} = \sqrt{-5k+2}$

2. $2\sqrt{2m+5} = \sqrt{4m+92}$

3. $\sqrt{m} + 2 = \sqrt{m+13}$

4. $\sqrt{x+4}+2=\sqrt{x+24}$

5. $\sqrt{x+7}+4=\sqrt{x+55}$

6. $\sqrt{2x-4}+2=\sqrt{3x+4}$