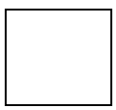
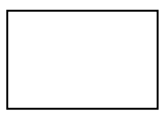
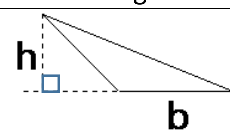
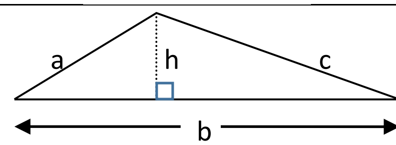
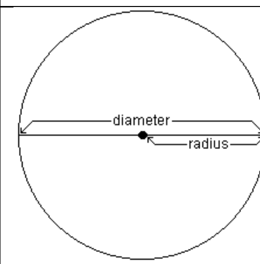
Applications of Linear Equations

Procedure for Solving Application Problems.

1. Read the problem carefully.
2. Determine the unknown and assign a variable to it. Other unknowns in the problem should be represented in terms of the same variable initially chosen.
3. Set up your equation.
4. Solve your equation.
5. Answer the question asked in a full sentence using appropriate units.

In this video the emphasis is on understanding how to set up the equation that models the word problem. In order to accomplish this many word problems need to be shown. To save time I will not spend time showing how to solve the equations. The solutions will be shown. Pause the video if you would like to go through the steps carefully.

Objective 1: Perimeter and Area Problems

	Square	Rectangle	Triangle	Circle
Learn these formulas	x  x	W  L	 	
Perimeter	$P = 4x$	$P = 2L + 2W$	$P = a + b + c$	Circumference $C = 2\pi r$ or $C = \pi d$
Area	$A = x^2$	$A = LW$	$A = \frac{1}{2}bh$	$A = \pi r^2$

Example: It will take 380 feet of fencing to fence in Gabriel's backyard. The length of his backyard is 50 feet less than twice the width. Find the dimensions of Gabriel's backyard.

$2(2w - 50) + 2(w) = 380$

$4w - 100 + 2w = 380$

$6w - 100 = 380$

$6w = 480$

$w = 80$

Width is 80

Length = 2(80) – 50 = 110

The dimensions of Gabriel's backyard are 80' by 110'.

Example: The perimeter of a triangle is 170 meters. One side is 40 more than the second side. The third side is 15 less than the first side. Find the lengths of the sides of the triangle.

$$(x)+(x-40)+(x-15)=170$$
$$x+x-40+x-15=170$$
$$3x-55=170$$
$$3x=225$$
$$x=75$$

First side = 75

Second side = 75 – 40 = 35

Third side = 75 – 15 = 60

The sides of the triangle are 75m, 35m and 60m.

Objective 2: Distance, Rate and Time Problems

Formula: Distance = Rate * Time

Example: Andy and Tommy start from the same point and walk in opposite directions. Andy walks twice as fast as Tommy does. In 7 hours they are 42 miles apart. How fast was Andy walking? How fast was Tommy walking?

$$(2r)(7)+(r)(7)=42$$
$$14r+7r=42$$
$$21r=42$$
$$r=2$$

Tommy = 2 m.p.h.

Andy = 2(2) = 4 m.p.h.

Tommy is walking at 2 m.p.h. and Andy is walking at 4 m.p.h.

Example: Carla and Joaquin went for a bike ride from their house to the beach. Carla rode at 15 m.p.h., while Joaquin rode at 11 m.p.h. It took Joaquin 1 hour longer to make it to the beach. How long did it take each of them to ride to the beach?

$$(15)(x)=(11)(x+1)$$
$$15x=11x+11$$
$$4x=11$$
$$x=\frac{11}{4}=2\frac{3}{4} \qquad x+1=3\frac{3}{4}$$

It took Carla $2\frac{3}{4}$ hours, while it took Joaquin $3\frac{3}{4}$ hours.

Example: On a 340-mile trip, a car traveled at an average speed of 55 m.p.h. and then reduced the speed to 40 m.p.h. for the remainder of the trip. The trip took a total of 7 hours. For how long did the car travel at each speed?

> If there are two unknowns in a problem that are supposed to add up to 7, then you can represent one as x and the other as 7 – x.

$$(55)(x)+(40)(7-x)=340$$
$$55x+280-40x=340$$
$$15x+280=340$$
$$15x=60$$
$$x=4 \qquad 7-x=3$$

The car travelled at 55 m.p.h. for 4 hours and at 40 m.p.h. for 3 hours.

Questions

Set up the equation that models the following word problem.

1. The length of a rectangle is 11 m more than twice its width. If the perimeter is 244 m, find the length and width of the rectangle.

2. Jack and Sally each drove from Buffalo, New York to Columbus, Ohio. Sally drove her truck at a rate of 65 mph. Jack drove his moped at a rate of 13 mph. It took Jack 8 hours longer than it did Sally. How long did it take each of them to make the trip?

3. Grover drove his car from Albany, New York to Concord, New Hampshire, a distance of 151 miles. He was driving at 45 mph. The weather got bad and he had to reduce his speed to 38.5 mph. The total trip took 3 hours. How long did he drive at each speed?

Objective 3: Value Mixture Problems

Amounts: If you take 3 gallons and add 2 gallons your new mixture must have 5 gallons in it.

If you take 3 gallons and add x gallons your new mixture must have (3 + x) gallons in it.

If you want to end up with 9 gallons in your final mix, then you would have x gallons of one mix and (9 – x) gallons of the other.

Formula: When you buy 2 pounds of hamburger that costs 3 dollars per pound, your bill will be $6.

$$\left(\frac{dollars}{pound}\right)(pounds) = (dollars)$$

Example: How many pounds of walnuts valued at $7.50 per pound should be mixed with 10 pounds of cashews valued at $9.00 per pound to create a trail mix valued at $8.00 per pound?

$$\left(\frac{dollars}{pound}\right)(pounds) + \left(\frac{dollars}{pound}\right)(pounds) = \left(\frac{dollars}{pound}\right)(pounds)$$

$$(\qquad)(\qquad)+(\qquad)(\qquad)=(\qquad)(\qquad)$$

$$(7.5)(x)+(9)(10)=(8)(x+10)$$

$$7.5x+90=8x+80$$

$$7.5x+10=8x$$

$$10=0.5x$$

$$10=\frac{1}{2}x$$

$$20=x$$

20 pounds of walnuts should be added.

Example: A chocolate shop manager mixes dark chocolate worth \$5.00 per pound with white chocolate worth \$9.00 per pound. Find how many pounds of each she should use to get 24 pounds of an assortment mix worth \$7.50 per pound.

$$\left(\frac{dollars}{pound}\right)(pounds) + \left(\frac{dollars}{pound}\right)(pounds) = \left(\frac{dollars}{pound}\right)(pounds)$$

$$(\qquad)(\qquad)+(\qquad)(\qquad)=(\qquad)(\qquad)$$

$$(5)(x)+(9)(24-x)=(7.5)(24)$$
$$5x+216-9x=180$$
$$-4x+216=180$$
$$-4x=-36$$
$$x=9 \qquad 24-x=15$$

9 pounds of the \$5 dark chocolate and 15 pounds of the \$9 white chocolate should be used.

Objective 4: Concentration Mixture Problems

Formula: If you have a bucket that has one gallon of a mix that is 50% antifreeze, then there is ½ gallon of antifreeze in the bucket.

50% of 1 gallon is 0.50 gallon.

$$(0.50)(1\ gallon) = 0.50\ gallon\ of\ \ pure\ antifreeze$$

(concentration)(amount) = amount of pure substance

Example: A chemist wants to make 50 ml of a 16% acid solution by mixing a 13% acid solution and an 18% acid solution. How many milliliters of each solution should the chemist use?

(concentration)(amount) + (concentration)(amount) = (concentration)(amount)

$$(\qquad)(\qquad)+(\qquad)(\qquad)=(\qquad)(\qquad)$$

$$(0.13)(x)+(0.18)(50-x)=(0.16)(50)$$
$$0.13x+9-0.18x=8$$
$$-0.05x+9=8$$
$$-0.05x=-1$$
$$x=\frac{-1}{-0.05}\cdot\frac{-100}{-100}=\frac{100}{5}=20 \qquad 50-x=30$$

20 ml of the 13% solution and 30 ml of the 18% solution should be used.

Example: Maya has 3 gallons of a solution that is 30 percent antifreeze, which she wants to use to winterize her car. How much pure antifreeze should she add to this solution so that the new solution will be 65 percent antifreeze?

(concentration)(amount) + (concentration)(amount) = (concentration)(amount)

$$(\quad)(\quad)+(\quad)(\quad)=(\quad)(\quad)$$

$$\begin{aligned}(0.3)(3)+(1)(x)&=(0.65)(3+x)\\ 0.9+x&=1.95+0.65x\\ 1x&=1.05+0.65x\\ (1-0.65)x&=1.05\\ 0.35x&=1.05\\ x&=\frac{1.05}{0.35}\cdot\frac{100}{100}=\frac{105}{35}=3\end{aligned}$$

3 gallons of the pure antifreeze should be added.

Example: Denise has 75 liters of a 30% salt solution. How many liters does she need to evaporate off to get a 35% salt solution?

(concentration)(amount) - (concentration)(amount) = (concentration)(amount)

$$(\quad)(\quad)-(\quad)(\quad)=(\quad)(\quad)$$

$$\begin{aligned}(0.3)(75)-(0)(x)&=(0.35)(75-x)\\ 22.5-0&=26.25-0.35x\\ 0.35x&=26.25-22.5\\ 0.35x&=3.75\\ x&=\frac{3.75}{0.35}\approx 10.7\end{aligned}$$

She would need to evaporate off approximately 10.7 liters.

Example: Bonnie has 50 gallons of a 20% solution. How many gallons must she drain off, then replace with the pure chemical to obtain a new mix that is 35% pure?

(concentration)(amount) - (concentration)(amount) + (concentration)(amount) = (concentration)(amount)

$$(\quad)(\quad)-(\quad)(\quad)+(\quad)(\quad)=(\quad)(\quad)$$

$$(0.2)(50)-(0.2)(x)+(1)(x)=(0.35)(50)$$

$$10-0.2x+x=17.5$$

$$0.8x=17.5-10$$

$$0.8x=7.5$$

$$x=9.375$$

She would need to drain off and replace 9.375 gallons.

Questions

Set up the equation that models the following word problem.

1. Gremlin's Mixed Nuts wants to create a mix of peanuts and cashews that costs $7.00 per pound. The peanuts they use cost $3.00 per pound. The cashews cost $9.00 per pound. How many pounds of each must be used to create a mix that weighs 12 pounds?

2. Valerie Stein, a pharmacist, wants to mix 150 liters of an 8% solution with a 16% solution. If she needs the mixture to be 10% alcohol, how much of the 16% solution must she add in?

3. Carla has 60 liters of a 27% salt solution. She wants to evaporate off enough to give her a 31% solution. How much does Carmen need to evaporate off?

Objective 5: Consecutive Integer Problems

Consecutive integers can be represented as: x, x+1, x+2, …

Consecutive odd integers can be represented as: x, x+2, x+4, …

Consecutive even integers can be represented as: x, x+2, x+4, …

Example: The sum of three consecutive integers is 48 more than twice the smallest. Find the integers.

$$(x)+(x+1)+(x+2)=48+2(x)$$
$$x+x+1+x+2=48+2x$$
$$3x+3=48+2x$$
$$x=45 \qquad x+1=46 \qquad x+2=47$$

The three integers are 45, 46 and 47.

Example: Find three consecutive odd integers such that the sum of 3 times the smaller and twice the largest is 143.

$$3(x)+2(x+4)=143$$
$$3x+2x+8=143$$
$$5x+8=143$$
$$5x=135$$
$$x=27 \qquad x+2=29 \qquad x+4=31$$

The three integers are 27, 29 and 31.

Objective 6: Problems Involving Fixed and Variable Costs

Example: One phone company charges $0.45 for the first minute and $0.20 per **minute after that**. If a phone call costs $2.85, how many minutes did the people talk? (don't forget to add in the first minute to your answer)

$$(0.45)+(0.20)(x)=2.85$$
$$0.45+0.2x=2.85$$
$$0.2x=2.85-0.45$$
$$0.2x=2.4$$
$$x=\frac{2.4}{0.2}\cdot\frac{10}{10}=\frac{24}{2}=12$$

The total phone call was 13 minutes long.

Questions

Set up the equation that models the following word problem.

1. The sum of three consecutive integers is 37 more than twice the largest. Find the integers.

2. Find three consecutive odd integers such that the sum of 5 times the smaller and 3 times the largest is 356.

3. Randy's Rent a Wreck rents trucks for $72 plus $0.80 per mile. Alex rented a truck last Monday. His bill was $147.20. How many miles did Alex drive the rented truck?

Objective 7: Problems Involving Complementary and Supplementary Angles

Angles are complementary if they add up to 90°.

Angles are supplementary if they add up to 180°

If you are working with two angles that are complementary you can represent them as x and 90 - x.

If you are working with two angles that are supplementary you can represent them as x and 180 - x.

Example: Find the measure of an angle if the measure of the angle is 8° less than three times the measure of its supplement.

$$\begin{aligned}(x) &= 3(180-x)-8\\ x &= 540-3x-8\\ 4x &= 532\\ x &= 133\end{aligned}$$

The measure of the angle is 133°.

Example: Find the measure of an angle whose supplement measures 6° more than 7 times its complement.

$$\begin{aligned}(180-x) &= 6+7(90-x)\\ 180-x &= 6+630-7x\\ 180-x &= 636-7x\\ -x+7x &= 636-180\\ 6x &= 456\\ x &= 76\end{aligned}$$

The measure of the angle is 76°.

Objective 8: Investment Problems

Simple interest = (principal)*(rate)*(time)

Example: Tyson invested $10,000 more at 8% than he did at 6%. His annual interest was $3,390 from the two investments. Find out how much he invested in each account.

$$(\text{principal})(\text{rate})(\text{time})+(\text{principal})(\text{rate})(\text{time})=\text{total interest}$$

$$\begin{aligned}(x+10000)(0.08)(1)+(x)(0.06)(1) &= 3390\\ 0.08x+800+0.06x &= 3390\\ 0.14x+800 &= 3390\\ 0.14x &= 2590\\ x &= 18500 \qquad x+10000=28500\end{aligned}$$

Tyson invested $18,500 at 6% and $28,500 at 8%.

Check: $(18500)*(.06)+(28500)*(.08)=3390$

Questions

Set up the equation that models the following word problem.

1. Find the measure of an angle if the measure of the angle is 58 less than three times the measure of its complement.

2. Find the measure of an angle if its supplement is 11 more than twice its complement.

3. George had $11,700 to invest. He chose to invest some of his money in a C.D. that earned 3% annual interest. The rest of his money he invested in a money market account that earned 5% annual interest. His total annual interest from the two accounts was $495. How much did George invest in each account?