

Completing the Square

Objective 1: Square Root Property

Quadratic Equations in Standard Form

Suppose a , b and c are real numbers such that $a \neq 0$.

$$ax^2 + bx + c = 0$$

Square Root Property

Suppose k is a real number. If $x^2 = k$, then $x = \sqrt{k}$ or $x = -\sqrt{k}$.

This may also be written as $x = \pm \sqrt{k}$.

Examples: Solve each quadratic equation.

a) $x^2 + 3 = -15$

Isolate the squared term.

Use the Square Root Property.

Remember that $i = \sqrt{-1}$.

Simplify the radical.

Solution set.

b) $(4m + 5)^2 - 8 = 20$

Isolate the squared term.

Use the Square Root Property.

Simplify the radical.

Solve for the variable.

Solution set.

Practice: Solve each quadratic equation by using the Square Root Property and write the solution set.

1. $2x^2 + 40 = 0$

2. $(3a+4)^2 - 7 = 11$

Objective 2: Perfect Square Trinomials

Perfect Square Trinomials

$$a^2 + 2ab + b^2 = (a + b)^2$$

or

$$a^2 - 2ab + b^2 = (a - b)^2$$

Examples: Factor the perfect square trinomial.

a) $x^2 + 18x + 81$

b) $9c^2 - 66c + 121$

Completing a Perfect Square Trinomial

We can make $x^2 + bx$ a perfect square trinomial by adding to it

$$\left[\frac{1}{2}b\right]^2$$

Example: Find what number must be added to the expression in order to make it a perfect square trinomial. Then factor the trinomial.

$$x^2 - 8x$$

Practice: Factor the perfect square trinomial.

1. $x^2 - 14x + 49$

2. $18m^2 + 90m + 25$

Practice: Find what number must be added to the expression in order to make it a perfect square trinomial. Then factor the trinomial.

3. $t^2 - 24t$

Objective 3: Completing the Square When $a = 1$

Example: Solve the quadratic equation by completing the square.

$$x^2 - 4x - 10 = 0$$

Since the leading coefficient is already 1, then isolate the variable terms.

Complete the perfect square trinomial.

Factor the perfect square trinomial and simplify the numbers.

Use the Square Root Property and solve for the variable.

Use solution sets.

Practice: Solve each quadratic equation by completing the square and write the solution set.

1. $x^2 + 16x + 57 = 0$

2. $x^2 - 5x - 1 = 0$

Objective 4: Completing the Square When $a \neq 1$

Example: Solve the quadratic equation by completing the square.

$$2x^2 + 3x - 52 = 0$$

Isolate the variable terms. Then divide every term by the leading coefficient a when $a \neq 1$.

Now complete the perfect square trinomial and solve for the variable.

Practice: Solve each quadratic equation by completing the square and write the solution set.

1. $5x^2 - 50x + 155 = 0$

2. $3x^2 + 8x + 6 = 0$