

Domain and Range

Objective 1: Relations, Functions, Domain and Range

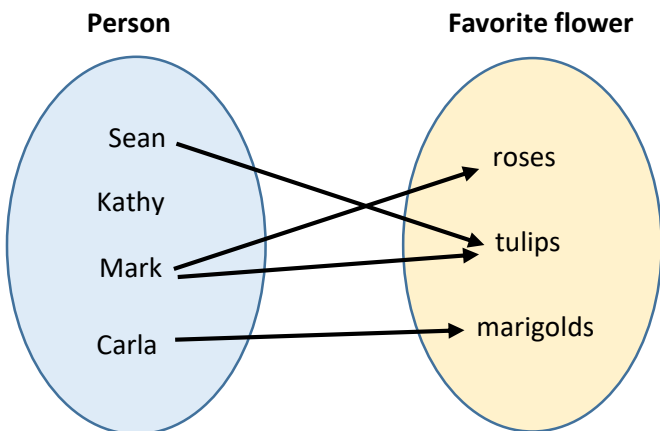
- A relation is a mapping from one set to another set.
- The first set is the domain.
- The second set is the range.
- A function is a special type of relation where each element in the domain is mapped to exactly one element in the range.

Example:

x	y
-2	3
0	6
1	-5
4	7

- -2 is mapped to 3, 0 is mapped to 6, 1 is mapped to -5 and 4 is mapped to 7.
- This relation is a function since each x value is mapped to exactly one y value.
- The domain is the set $\{-2,0,1,4\}$ since these are all of the possible x values.
- The range is the set $\{-5,3,6,7\}$ since these are all of the possible y values.

Example:



- This relation is not a function for two reasons
 - Kathy does not have a favorite flower. All elements in the domain must map to an element in the range.
 - Mark has two favorite flowers. So, this x value is mapped to more than one y value.
- Domain = {Sean, Kathy, Mark, Carla}
- Range = {roses, tulips, marigolds}

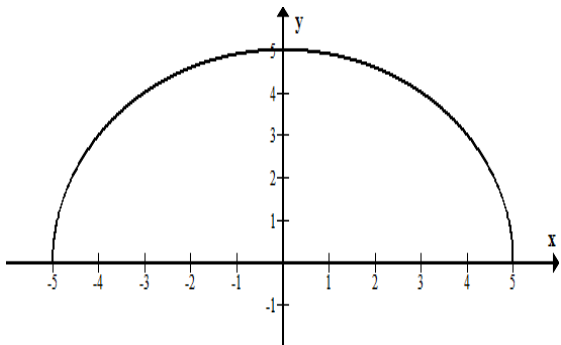
Example:

$\{(-2,3), (0,2), (1,2)\}$

- This relation is a function.
 - Note that there are two different x values that map to one y value. That is fine. You just can't have one x value map to more than one y value.
- Domain = $\{-2,0,1\}$
- Range = $\{2,3\}$

Objective 2: Determine the Domain and Range of a Relation from the Graph

Example:



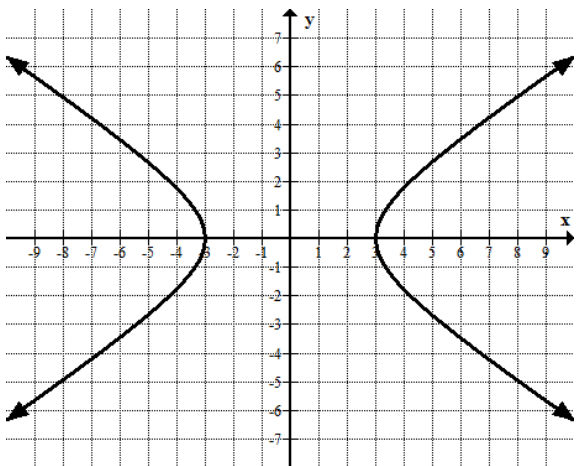
- This relation is a function, since each x value maps to exactly one y value. The way that you can tell by looking at the graph is if no matter where you draw a vertical line on the graph, you can NOT touch more than one of the points on the graph (vertical line test), then it is a function.
- Domain is $[-5, 5]$, which can also be written as $\{x \mid -5 \leq x \leq 5\}$. From the graph you look at how far left and how far right the graph goes, and if there are any breaks in the graph.
- Range is $[0, 5]$, which can also be written as $\{y \mid 0 \leq y \leq 5\}$. From the graph you look at how far down and how far up the graph goes, and if there are any breaks in the graph.

It is helpful to know both set builder notation and interval notation.

Graph			
Set builder notation	$\{x \mid -2 < x \leq 5\}$	$\{x \mid x > 3\}$	$\{x \mid x \leq 3\}$
Interval notation	$(-2, 5]$	$(3, \infty)$	$(-\infty, 3]$

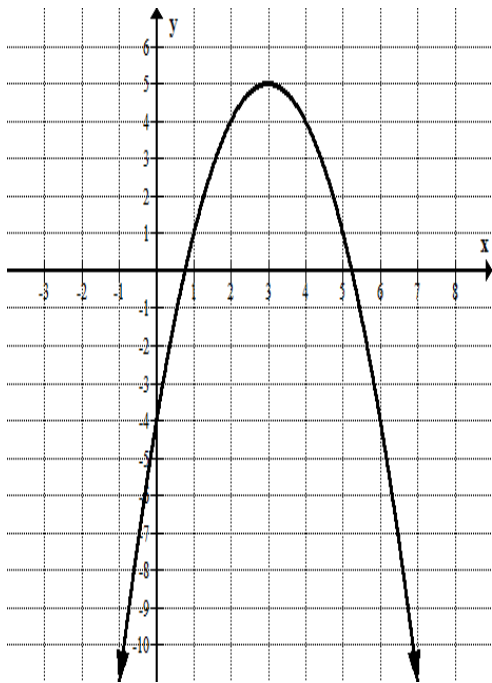
Graph		
Set builder notation	$\{x \mid x \neq 3\}$	$\{x \mid x \leq 1 \text{ or } x > 7\}$
Interval notation	$(-\infty, 3) \cup (3, \infty)$	$(-\infty, 1] \cup (7, \infty)$

Example:



- Is this relation a function? No
- Domain = $\{x \mid x \leq -3 \text{ or } x \geq 3\}$ or $(-\infty, -3] \cup [3, \infty)$
- Range = All real numbers or $(-\infty, \infty)$

Example:



- a) Is this relation a function? Yes
- b) Domain = All real numbers or $(-\infty, \infty)$
- c) Range = $\{y \mid y \leq 5\}$ or $(-\infty, 5]$

Objective 3: Determine the Domain of a Function from the Equation

$$f(x) = \frac{4}{x}$$

What is $f(4)$?

What is $f(2)$?

What is $f(0)$?

$$g(x) = \frac{9}{x-3}$$

What is $g(0)$?

What is $g(3)$?

If you are given an equation that defines a function and are not told any restrictions on the domain, then the domain is the set of all real numbers for which the expression is defined. So, we ask, when would it not be defined?

1. We know that we can not divide by zero.
2. We know that we can not take the square (or any even) root of a negative number.

These are the two restrictions we must look for. The domain is all OTHER real numbers.

Example: $f(x) = \frac{x+2}{x-3}$

1. Set the denominator equal to zero: $x-3=0$ so $x=3$
2. There are no square roots to worry about.

Domain

Set builder notation: $\{x \mid x \neq 3\}$

Interval notation: $(-\infty, 3) \cup (3, \infty)$

Example: $f(x) = \sqrt{3x-2}$

1. There is no denominator to deal with.
2. What is inside the square root (called the radicand) must be ≥ 0 . (recall $\sqrt{0} = 0$ is fine)

$$3x - 2 \geq 0$$

$$3x \geq 2$$

$$x \geq \frac{2}{3}$$

Domain

Set builder notation: $\left\{x \mid x \geq \frac{2}{3}\right\}$

Interval notation: $\left[\frac{2}{3}, \infty\right)$

Example: $f(x) = 2x^3 - 4x + 7$

1. There is no denominator to deal with.
2. There are no square roots to worry about.

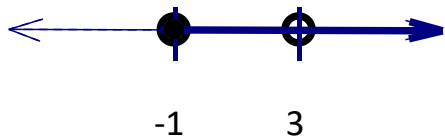
Domain = all real numbers or $(-\infty, \infty)$

Example: $f(x) = \frac{\sqrt{x+1}}{x-3}$

1. Set the denominator equal to zero: $x-3=0$ so $x=3$
2. What is inside the square root (called the radicand) must be ≥ 0 .

$$x+1 \geq 0 \quad \text{and} \quad x \neq 3$$

$$x \geq -1$$



Domain

Set builder notation: $\{x \mid -1 \leq x < 3 \text{ or } x > 3\}$

Interval notation: $[-1, 3) \cup (3, \infty)$