Radicals

Objective 1: Taking n^{th} Roots with Numbers

n^{th} Roots

b is the n^{th} root of a if $b^n = a$. We use the following notation.

$$b = \sqrt[n]{a}$$

- b is the n^{th} root
- n is the index
- a is the radicand
- $\sqrt[n]{a}$ is the radical

Perfect square numbers:		Perfect square roots:
$1^2 = 1$	$9^2 = 81$	$\sqrt{1} =$
$2^2 = 4$	$10^2 = 100$	$\sqrt{4} =$
$3^2 = 9$	$11^2 = 121$	$\sqrt{9} =$
$4^2 = 16$	$12^2 = 144$	$\sqrt{16} =$
$5^2 = 25$	$13^2 = 169$	$\sqrt{25} =$
$6^2 = 36$	$14^2 = 196$	
$7^2 = 49$	$15^2 = 225$	
$8^2 = 64$	$25^2 = 625$	

Perfect cubes:	Perfect cube roots:
$1^3 = 1$	$\sqrt[3]{1} =$
$2^3 = 8$	$\sqrt[3]{8} =$
$3^3 = 27$	$\sqrt[3]{27} =$
$4^3 = 64$	$\sqrt[3]{64} =$
$5^3 = 125$	$\sqrt[3]{125} =$
$6^3 = 216$	
$7^3 = 343$	

Example: Simplify the following radical.

$\sqrt{49} =$

To be a **perfect square**, a number must have **two** of the same factor. When this happens, we can take those two factors together, and allow **one** to ``escape'' the **square root**.

$\sqrt[3]{216} =$

To be a **perfect cube**, a number must have **three** of the same factor. When this happens, we can take those three factors together, and allow **one** to ``escape'' the **cube root**.

Principal and Negative Square Roots

If a is a nonnegative number, then

1. $\sqrt{a}$ is the **principal,** or **nonnegative square root** of a

2. $-\sqrt{a}$ is the **negative square root** of a

3. The square root of a negative number is not a real number. $\sqrt{-9}$ is not a real number.

4. $\sqrt{0} = 0$ and $\sqrt{1} = 1$

If the index is even, such as $\sqrt{}$, $\sqrt[4]{}$, or $\sqrt[6]{}$, the radicand must be nonnegative for the root to be a real number.

For example: $\sqrt[4]{16} = 2$, but $\sqrt[4]{-16}$ is not a real number.

$\sqrt[6]{64} = 2$, but $\sqrt[6]{-64}$ is not a real number.

If the index is odd, such as $\sqrt[3]{}$, $\sqrt[5]{}$, or $\sqrt[7]{}$, the radicand may be any real number. For example:

$\sqrt[3]{64} = 4$, and $\sqrt[3]{-64} = -4$

$\sqrt[5]{32} = 2$, and $\sqrt[5]{-32} = -2$

If we do not get a perfect power that matches the index of the radical, we can still take out any double-factors that exist, and whatever is left over will be stuck under the radical.

<u>Example:</u>

$\sqrt{18}$

$\sqrt[3]{40}$

$\sqrt{162}$

Objective 2: Taking n^{th} Roots with Variables

Variables follow the same rules that numbers do: It takes 2 factors to escape the square root, 3 factors to escape the cube root, etc.

Perfect squares:	Perfect square roots:
$(x)^2 = x^2$	$\sqrt{x^2} =$
$(x^2)^2 = x^4$	$\sqrt{x^4} =$
$(x^3)^2 = x^6$	$\sqrt{x^6} =$
$(x^4)^2 = x^8$	$\sqrt{x^8} =$
$(x^5)^2 = x^{10}$	$\sqrt{x^{10}} =$

Perfect cubes:	Perfect cube roots:
$(x)^3 = x^3$	$\sqrt[3]{x^3} =$
$(x^2)^3 = x^6$	$\sqrt[3]{x^6} =$
$(x^3)^3 = x^9$	$\sqrt[3]{x^9} =$
$(x^4)^3 = x^{12}$	$\sqrt[3]{x^{12}} =$
$(x^5)^3 = x^{15}$	$\sqrt[3]{x^{15}} =$

Examples: Simplify the following radicals. Assume variables represent positive real numbers.

a) $\sqrt{x^4}$

b) $\sqrt{x^{10}}$

c) $\sqrt{x^{11}}$

d) $\sqrt[3]{x^{17}}$

e) $\sqrt[5]{x^{23}}$

f) $\sqrt[4]{3^5 \cdot 5^6 \cdot x^{11} y^{12}}$

g) $\sqrt[5]{96a^{45}b^{37}}$

Objective 3: Adding and Subtracting Radicals

"Like radicals" have the same index and the same radicand.

We add or subtract radicals similar to the way we "combine like terms."

Add or subtract the coefficients and keep the radical.

Polynomials	***Radicals***
$3x + 7x$ $= (3+7)x$ $= 10x$	$3\sqrt{5} + 7\sqrt{5}$ $= (3+7)\sqrt{5}$ $= 10\sqrt{5}$

If the radicals are not the same, then we will simplify them first before we combine like terms.

Examples:

a) $4\sqrt{7} - 9\sqrt{7}$

b) $9\sqrt[3]{2y} + 15\sqrt[3]{2y}$

c) $\sqrt{50} + 5\sqrt{18}$

d) $\sqrt{20x} - 6\sqrt{16x} + \sqrt{45x}$

e) $\sqrt[3]{8y^5} + \sqrt[3]{27y^5}$

Objective 4: Rationalizing Monomial Denominators

"Simplified Form" for Radicals $\sqrt[n]{}$

1) no perfect nth powers in the radicand

2) no fractions in the radicand

3) no radicals in the denominator

Product Rule for Radicals $\sqrt[n]{a} \bullet \sqrt[n]{b} = \sqrt[n]{ab}$

Quotient Rule for Radicals $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Note: we can use these rules in "either direction."

<u>Rationalizing the Denominator of a Fraction (one term in the denominator)</u>

The process of eliminating the radical in the denominator is called rationalizing the denominator.

If the denominator is a single term, we multiply numerator and denominator so that the denominator is a perfect square, or perfect cube, or perfect 4th root, etc. depending on the index of the radical.

<u>**Examples**</u>: Simplify. Since each of these expressions have a radical in the denominator, simplifying includes rationalizing the denominator. Assume variables represent positive real numbers.

a) $\dfrac{2}{\sqrt{5}}$

b) $\sqrt[3]{\dfrac{1}{5}}$

c) $\dfrac{6}{\sqrt[4]{x}}$

d) $\dfrac{8}{\sqrt[7]{x^5}}$

e) $\dfrac{\sqrt[5]{a^2}}{\sqrt[5]{32b^{17}}}$

f) $\dfrac{\sqrt{18}}{\sqrt{75}}$

Objective 5: Rationalizing Binomial Denominators

Recall from factoring, the formula $(a+b)(a-b) = a^2 - b^2$. This formula plays an integral part in allowing us to rationalize if our denominator is a binomial involving square roots, where $(a+b)$ and $(a-b)$ are called conjugates of each other.

Example: Multiply the following, then simplify.

$\left(4-\sqrt{3}\right)\left(4+\sqrt{3}\right)$

Examples: Simplify. Since each of these expressions have a radical in the denominator, simplifying includes rationalizing the denominator. Assume variables represent positive real numbers.

a) $\dfrac{6}{\sqrt{5}+\sqrt{3}}$

b) $\dfrac{3\sqrt{6}}{5\sqrt{3}-4\sqrt{2}}$

c) $\dfrac{\sqrt{x}+1}{\sqrt{x}-10}$