

Solving Quadratic Equations

There are four methods for solving quadratic equations.

1. Solve by taking square roots (also called extracting square roots).
2. Solve by factoring – using the zero product rule.
3. Solve by completing the square.
4. Solve using the quadratic formula.

Objective 1: Solving by taking square roots

Isolate the perfect square, take the square root of both sides, solve.

Example: $2x^2 - 3 = 7$

Example: $2(x+4)^2 - 5 = 19$

Objective 2: Solve by factoring

Move all terms to one side, factor, set each factor equal to zero, solve.

Example: $5x^2 + 8 = -14x$

Example: $15x^2 - 14x = 8$

Objective 3: Solve by completing the square

Move the constant term to the other side, divide by the leading coefficient so that you have $1x^2$, complete the square, take square roots, then solve.

Example: $x^2 - 10x - 3 = 0$

Example: $4x^2 + 24x - 8 = 0$

Objective 4: Solve by using the quadratic formula

Move all terms to one side, determine a, b and c, write down the formula, substitute in your values, then simplify the result.

Example: $3x^2 + 7 = -12x$

Example: $5x^2 = 8x + 3$

Objective 5: Determine the best method to solve a quadratic equation

In the equation $ax^2 + bx + c = 0$

1. ax^2 is the quadratic term
2. bx is the linear term
3. c is the constant

If your equation **does not have a constant**, then solve by factoring.

$$3x^2 - 6x = 0 \quad \text{All nonzero terms have } x, \text{ no constants.}$$

$$3x^2 - 6x = 0$$

$$3x(x-2) = 0$$

$$3x = 0 \text{ or } x - 2 = 0$$

$$x = 0 \text{ or } x = 2$$

Any time the equation factors easily, this is the easiest method.

If your equation **does not have a linear term**, then solve by taking square roots.

$$4x^2 - 9 = 0$$

Note that there is a quadratic term and a constant, but no linear term.

$$4x^2 = 9$$

$$x^2 = \frac{9}{4}$$

$$x = \pm \frac{3}{2}$$

If your equation looks like $1x^2 + bx + c = 0$ and **b is even**, then completing the square is really fast.

$$x^2 - 8x = 3$$

$$x^2 - 8x + 16 = 3 + 16$$

$$(x - 4)^2 = 19$$

$$x - 4 = \pm\sqrt{19}$$

$$x = 4 \pm \sqrt{19}$$

If your equation looks like:

$3(x+2)^2 - 4 = 8$ then solve by taking square roots.

$$3(x+2)^2 = 12$$

$$(x+2)^2 = 4$$

$$x+2 = \pm\sqrt{4}$$

$$x+2 = \pm 2$$

$$x = -2 + 2 \text{ or } x = -2 - 2$$

$$x = 0 \text{ or } x = -4$$

If your equation involves **fractions**, multiply both sides of the equation by the appropriate quantity to **get rid of** the fractions.

If your equation involves **decimals**, then the quadratic formula is probably the easiest method.

If you are doing a **word problem** or any problem where you are allowed to use an **approximation**, the quadratic formula is usually fastest. Substitute a, b and c into the formula, then pull out a calculator and approximate the answers – no need to simplify.

Objective 6: Practice solving by any method.

Choose the easiest method, then solve each of the following quadratic equations