

Cypress College Math Review: Variation of Parameters

We wish to solve second order nonhomogeneous equations $a_0(x)y'' + a_1(x)y' + a_2(x)y = F(x)$ where $F(x)$ is not a linear combination of functions of the type $x^j e^{ax} \cos bx$, $x^j e^{ax} \sin bx$, $0 \leq j \leq k-1$ in which the method of undetermined coefficients would work well.

Let u_1, u_2 be linearly independent solutions to the homogeneous equation $Ly=0$.

$y_c = c_1 u_1(x) + c_2 u_2(x)$ be the general solution to the homogeneous equation.

Let $y_p = g_1(x)u_1(x) + g_2(x)u_2(x)$ be a particular solution to the nonhomogeneous equation $Ly=F(x)$ where g_1 and g_2 are obtained by solving the system of equations:

$$g_1' u_1' + g_2' u_2' = \frac{F(x)}{a_0}$$

$$g_1' u_1 + g_2' u_2 = 0$$

(I use g_1 and g_2 instead of the more standard C_1 and C_2 since those look too much like constants to me.)

Example) $y'' - 2y' + y = \frac{e^x}{2x}, x > 0$

Example) $y'' + y = \tan x$

Example) $(D^2 - 3D + 2)y = \frac{e^{2x}}{1 + e^{2x}}$